Targeted Information Design*

Shaping monopolistic markets with heterogeneous consumers

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A monopolistic seller offers a product through a platform to partially informed consumers. The platform observes both consumers' preferences and their existing information and decides how much additional information to disclose. I consider different objectives for the platform and show how the platform designs disclosure to shape the elasticity of demand depending on its objective. I characterize the set of feasible welfare outcomes and show that total surplus is maximized when the platform maximizes consumer surplus and decreases whenever the platform pursues any other objective, such as profit maximization.

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1 Introduction

In many markets, potential consumers only have partial information about the product they are considering to buy. Information provision therefore plays a central role in shaping trade outcomes: as consumers learn more, they make better purchasing decisions, directly affecting welfare. When such learning occurs at scale, it alters the structure of market demand, prompting the seller to adjust their prices. Nowadays, online platforms are an important source of information to consumers. By analysing data from past interactions, browsing histories, and consumer characteristics, platforms can provide information not available elsewhere. Through personalized recommendations and targeted advertising, they influence consumers' beliefs and, ultimately, market outcomes. Yet, platforms are rarely the sole source of information. Instead consumers typically already have some understanding of their own preferences when engaging with the platform. This information may be the result of consulting independent online reviews, learning from prior purchases etc. In addition, platforms often do not just hold information about consumers' preferences, but also regarding what consumers already know. Equipped with such knowledge platforms are able to engage in *targeted information design* by tailoring information provision to consumers knowledge outside their control. A natural question is then *how* platforms engage in such targeting and —more generally— *how* they provide information in the presence of outside information. While there is an extensive literature investigating how information provision shapes market outcomes, it is standard within this literature to focus on information designers with *full control*. The designer can decide on any level of information provision ranging from no to full information implying that the designer is the only source of information to consumers. Instead, I allow consumers to arrive to the market partially informed.

In my model one seller sells a good to consumers through a platform. The consumer is partially informed, but still uncertain about their preferences for the offered product. The platform decides how to provide additional information to the consumer by committing to a policy that determines how consumers receive information conditional on their true valuations and on outside information. Prices are set by the seller. The seller knows how information is provided to consumers but cannot condition prices on the realised recommendations. In this context, I study how a platform should provide information to consumers. I consider different objectives for the platform including seller profit and consumer surplus maximization and ask how total welfare changes with the platforms objective.

Allowing for the possibility of outside information, however, raises a few methodological challenges. In particular, accounting for this important feature of the environment, cannot be modelled as a mere relabelling of the state space. It is not enough to treat consumer valuation and outside information as a new state and then apply standard techniques since this overlooks the structure of the problem. In my setting, the platform observes the consumer's realized outside information, whereas the seller does not. This asymmetry links the information design problems across consumers with different outside information: the platform maximizes over the overall induced demand function rather than rather solving separate information design problems for each consumer type.

In principle, this requires solving an optimization problem with a potentially infinite-dimensional constraint—each posterior under outside information must be more informative after the platform's disclosure. Building on Ennuschat (2025)[1], I simplify this problem under an assumption on the structure of outside information that ensures posteriors are ordered and non-intersecting. Under this condition, the platform's problem can be reformulated as an optimization over the induced demand function.

With this, I study the platforms optimal disclosure rule for different objectives. A platform aiming to maximize revenue will design information that makes demand inelastic across a wide range of valuations, enabling the seller to extract more surplus. By contrast, a platform maximizing consumer surplus induces demand that is unit-elastic over some range of valuations, leaving the seller indifferent among multiple prices. This parallels Roesler and Szentes (2017), who characterize buyer-optimal information when no outside information exists. They focus on characterizing buyer optimal disclosure since absent outside information, the seller extracts all surplus without any revelation. Then, along the Pareto Frontier, the design of information only governs the distribution of welfare but not its level.

When outside information is present, however, efficiency varies along the Pareto frontier; moving toward the seller-optimal outcome reduces total welfare. Because

[1] Draft available upon request.

outside information alters the set of feasible demand functions that the platform can induce—rather than merely relabelling the state space—the resulting welfare implications cannot be replicated by a simple redefinition of the state. Accounting for such information qualitatively alters the insights obtained in information-design problems.

To understand this misalignment of total surplus and profits, take the extreme objective where the platform maximizes the seller's profit. For each possible price, consider the maximum quantity demanded the platform can generate through disclosure. Then disclosure reveals additional information to consumers, who, based solely on their outside information, would choose not to buy. As the price rises, this maximal demand necessarily falls, since convincing consumers to purchase by revealing information becomes harder. But the seller still benefits from higher prices if the resulting revenue gain outweighs the loss in demand. Yet, the reduction in trade means that some mutually beneficial transactions no longer occur leading to lower efficiency. Models studying market segmentation, which can be thought of as information revelation to the seller instead of the buyer, emphasize that, consumer optimal segmentation does not need to come at a cost of efficiency Bergemann et al. (2015); Haghpanah and Siegel (2023); Bergemann et al. (2025a). However, under outside information and when information is revealed to consumers, I establish alignment in a stronger sense: total surplus decreases as we move away from consumer towards seller-optimal disclosure.

The increasing importance of data as an economic resource has renewed interest in the effects of information provision on market outcomes. A plethora of papers investigates this question under different assumptions on the market structure and varies which side of the market receives information (Roesler and Szentes, 2017; Bergemann et al., 2015; Haghpanah and Siegel, 2023; Ravid et al., 2022; Bergemann et al., 2025a; Armstrong and Zhou, 2022; Elliot et al., 2025). A maintained assumption of these papers is that information is only provided by a single source. Results allowing for privately informed receivers are more sparse and often need additional assumptions on the environment to ensure tractability (Kolotilin et al., 2017; Guo and Shmaya, 2019; Candogan and Strack, 2023; Guo et al., 2025).

With respect to the literature, this paper makes three points. Instead of considering *full control* information design, it introduces an information environment, where consumers hold outside information which is not controlled by but observable to the platform. This setting reflects digital markets, where reviews or browsing histories are public and platforms retain the ability to condition disclosure on them. Second, it shows that allowing for outside information reveals a misalignment of total surplus and profit maximization. Third, it applies methodological insights from Ennuschat (2025)[2] to an economically relevant environment demonstrating how these methods yield tractable welfare comparisons when outside information shapes the feasible set of demand functions.

The remainder of this paper proceeds as follows. Section 2 introduces the model. In section 3 I characterize seller and buyer optimal learning respectively as well as the set of welfare outcomes. I also consider environments where outside information is not monotone non-overlapping and shows that the relaxed program characterizes a robust version of the set of welfare outcomes in general environments. Section 4 reviews the literature and Section 5 concludes.

2 Model

I consider a simple model of trade intermediated by a platform where a single buyer and seller interact through a platform. The buyer's valuation for the offered good is uncertain and imperfectly known to her. Before purchasing, the buyer receives additional information from the platform, which strategically decides how much information to provide. The seller produces at zero marginal cost and chooses a price after observing *how* the platform provides information to consumers.

Buyer's Purchasing Problem The buyer (she) is risk-neutral with valuation $v \in V = [0, 1]$ where $v \sim F_0$. She is partially informed of her valuation. Her outside information is represented by the signalling structure $\mathcal{S}^O = (S^O, \sigma^O)$ where $\sigma^O : V \to \Delta(S^O)$ is a mapping from valuations into (possibly random) signal realisations. A signalling structure $\mathcal{S}^O$ can equivalently be described by the distribution over posterior beliefs $\tau^O \in \Delta(\Delta(V))$ it induces (Blackwell, 1953; Terstiege and Wasser, 2023). A posterior $q^O \in \Delta(V)$ describes the probability the agent attaches to different valuations after seeing the signal realisation s^O. Then each signal realisation $s^O \in S^O$ induces a posterior q^O and the probability of each

[2] Draft available upon request.

posterior is given by the probability of the signal realisation that induces it.[3] Due to risk-neutrality, the buyers purchasing decision only depends on the mean of her posterior q^O, denoted by $\mathbb{E}_{v \sim q^O}(v) = \mu_{q^O}$. The probability distribution over posteriors, τ^O, induces a distribution over those posterior means, $G^O \in \Delta(V)$.

I impose the following assumption on the outside information:

Assumption 1. Outside information is minimal*: $\mathcal{S}^O$ is the unique least informative signalling structure inducing the posterior mean distribution G^O. Equivalently $\mathcal{S}^O$ is a garbling of any $\mathcal{S}^{O\prime}$ that also induces G^O.*[4]

When outside information is minimal, any other signalling structure that induces the same law of posterior means is Blackwell more informative. Such a non-minimal structure conveys extra detail beyond the posterior mean which is payoff-irrelevant to the buyer. Intuitively, you can think of minimal outside information as a "mean-only" signalling structure: it reveals the posterior mean, the buyer's payoff-relevant statistic, and nothing else.

To build better understanding of what minimality entails the following provides an equivalent geometric characterization in terms of distribution of posteriors τ that will be used later one (Ennuschat, 2025).

Definition 1 (Monotone Non-Overlapping Experiment)**.** *An experiment is monotone non-overlapping if*

i) τ *is convex: for all* $q \in supp(\tau)$ *we have that* $v \in supp(q)$ *whenever* $v \in \mathrm{co}(supp(q))$[5] *for all* $v \in V$

ii) τ *is non-overlapping: for all* $q, q' \in supp(\tau)$ *such that* $q \neq q'$ *we have that*

$$supp(q) \cap supp(q') \subseteq ext(q) \cap ext(q')$$

where $ext(q) = \{min(supp(q)), max(supp(q))\}$

A signalling structure corresponds to a monotone non-overlapping experiment if (i) the support of each posterior is convex and (ii) whenever the buyer attaches

[3] Blackwell (1953) shows the equivalence for finite state spaces and Terstiege and Wasser (2023) show this continues to hold for infinite state spaces.

[4] A signalling structure $\mathcal{S}'$ is a garbling of $\mathcal{S}$, if there exists a Markov kernel $\gamma : S \to \Delta(S')$

$$\sigma'(s' \mid \omega) = \int_{s \in S} \gamma(s' \mid s)\sigma(s \mid \omega)ds$$

[5] Let $\mathrm{co}(A)$ denote the convex hull of the set A.

positive probability to some valuation under multiple posterior, the valuation must always be either the highest or lowest valuation the buyer conceives of. Two important special cases are (i) when valuations are binary (then any signalling structure is monotone non-overlapping) and (ii) when outside information is monotone partitional.[6] Monotone partitional signalling structures are commonly encountered in the literature (Onuchic and Ray, 2023; Mensch, 2021; Ivanov, 2021). They are practical, easy to implement and optimal in many information design problems (Kolotilin et al., 2024; Dworczak and Martini, 2019). This kind of coarse, tiered information is natural in many settings. Consider a good that comes in a base specification and has tiered add-ons, meaning that a *premium* product will have all features a *plus* product has. A monotone partitional signalling structure then tells the buyer whether the good comes in "*base*", "*plus*" or "*premium*" specification.

Platform's Information Policy The platform (it) intermediates trade by providing information to the buyer. It can condition her information policy on the valuation v[7] as well as the realised outside information s^O. Notice that when outside information is monotone partitional, the second requirement becomes redundant: by knowing the buyer's valuation the platform knows the signal realisation. Formally, the platform chooses a signalling structure:

$$\mathcal{S}^D = (S^D, \sigma^D) \quad \text{where } \sigma^D : V \times S^O \to \Delta(S^D)$$

In words, σ^D maps valuations and signal realisations under outside information into (possibly random) messages. As before this can equivalently be described by the distribution over posterior beliefs $\tau^D \in \Delta(\Delta(V))$. Let μ_{q^D} denote the mean of posterior q^D and let $G^D \in \Delta(V)$ denote the induced posterior mean distribution.

A platform designing information cannot freely induce any distribution τ^D. Instead, it is constrained by the existence of outside information as well as her own knowledge: the buyer must be at least as informed as they were under outside information and at most fully informed.

By Blackwell's Theorem, a signal structure $\mathcal{S}^D$ is more informative than a signal structure $\mathcal{S}^O$ iff the distribution of posterior beliefs τ^D is a mean-preserving spread

[6]Outside information is *monotone partitional* whenever $\exists P : [0,1] \to [0,1]$ where P is weakly increasing and we have that $s^O(v) = s^O(v')$ whenever $P(v) = P(v')$ while $s^O(v) \neq s^O(v')$ if $P(v) \neq P(v')$.

[7]The analysis would equally go through when the platform can provide more accurate information at every posterior under outside information.

of the distribution of posterior beliefs τ^O.[8] On the other extreme, the platform can at most fully reveal what she knows about the buyers' valuation. Let F_0^δ denote the distribution over degenerate beliefs induced by full revelation. A signal structure $\mathcal{S}^D$ is less informative than full information, iff F_0^δ is a mean preserving spread the distribution of posterior beliefs mean-preserving spread of τ^D.

These two feasibility constraints can be understood as follows. Think of outside information τ^O as a two-stage lottery, where the first stage describes the probabilities of different posteriors and the second stage describes the probabilities attached to different valuations under the respective posterior. Additional revelation by the platform splits each posterior q^O—the second stage lottery— into multiple new posteriors. We now have a three-stage lottery where the compound lottery of second and third stage need to correspond to q^O. This is exactly what imposing that τ^D is a mean-preserving spread of τ^O requires. Conversely, we can think of providing less information as collapsing multiple posteriors into one: if τ^D is less informative than full revelation, F_0 is a mean-preserving spread of τ^D

The platform's objective is to maximize some function of expected consumer surplus and profits $f(\pi, CS)$ through the choice of a distribution over posteriors τ^D under the constraint that τ^D can be induced by some information policy. The platforms program is then given by :

$$\max_{\tau^D} f(\pi, CS) \qquad \text{s.t} \quad F_0^\delta \underset{MPS}{\geq} \tau^D \underset{MPS}{\geq} \tau^O$$

where $\underset{MPS}{\geq}$ denotes the partial order of mean preserving spreads.

Seller's Price Setting Problem The seller (he) sets prices.[9] When choosing prices the seller knows the signalling structures $\mathcal{S}^O$ and $\mathcal{S}^D$ but does not observe its signal realisations s^O and s^D. In other words he only knows *how* information is provided to the buyer but not what is learned. Consequently, the seller optimizes with respect to the expected demand of the buyer, given by all buyers whose expected

[8]A distribution of posterior beliefs $\tau^D \in \Delta(\Delta(V))$ is a mean-preserving spread of another distribution $\tau^O \in \Delta(\Delta(V))$ if there exist $\in \Delta(V)$-valued random variables q^D, q^O such that (i) $q^D \sim \tau^D$ and $q^O \sim \tau^O$ and (ii) $\mathbb{E}(q^D \mid q^O) = q^O$

[9]Notice that in this setting it will be without loss to focus on price posting as a mechanisms.

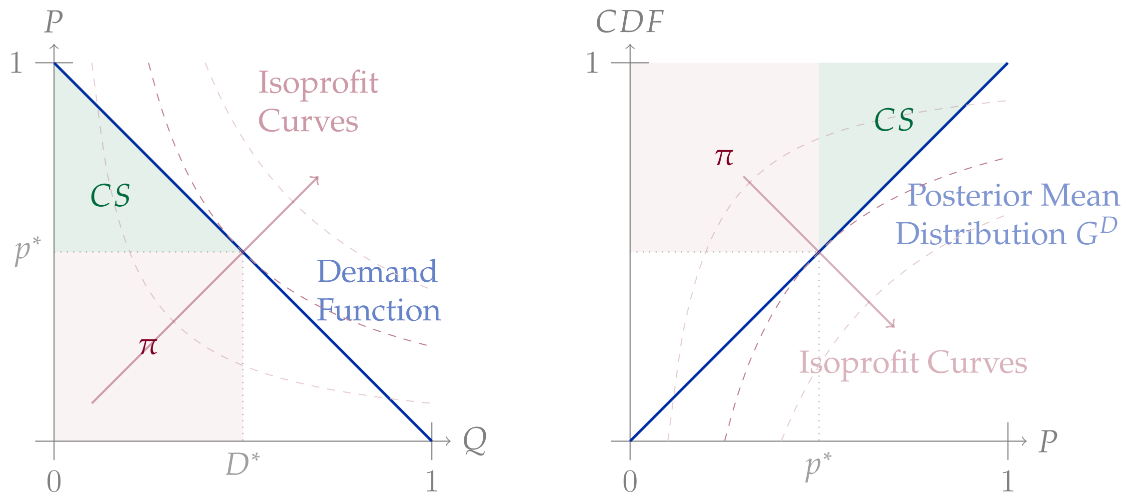

Figure 1: Sellers Optimal Price Setting Strategy

This figure demonstrates the seller's best-response when G^D is a uniform distribution over $[0, 1]$. The seller would set price of 0.5 and demand would be 0.5 leading to profits of 0.25.

valuation (post-learning) weakly exceeds the price:

$$D(G^D, p) = (1 - G^D(p)) + \Delta(G^D, p))$$

where $\Delta(G^D, p)$ denotes any potential mass of G^D at p. The seller maximizes profits:

$$\max_p \pi(p, G) = \max_p \left(1 - G\left(p\right) + \Delta\left(G, p\right)\right) p$$

To visualize the seller's maximization problem, consider isoprofit curves, describing all combinations of prices and demand that lead to the same level of profits, plotted as dashed red lines in Figure 1. For smooth demand functions, the seller sets a price so that the demand function is tangential to the highest isoprofit curve.

The right hand side illustrates how we can express the same logic in terms of CDF's of posterior mean distributions— a $90°$ degree rotation of the left panel. The red arrow indicates the direction in which profits increase: demand rises downward, price rightward. For a smooth CDF, the seller sets a price where the furthest south-east isoprofit curve is tangential to the CDF of posterior means G^D. This visually corresponds to maximizing profits.

Thus, the pricing decision of the seller depends on the shape of the posterior mean CDF induced by the platform's information policy. With this, we can turn to understand ow the platform leverages its ability to shape demand.

3 Results

This section develops the welfare and pricing implications of information provision by the platform . To do so, I start by introducing a relaxed version of the platform's optimization program to provide a tractable benchmark. I show that this relaxation is without loss under maintained assumptions. Using this relaxation, I turn to characterizing buyer- and seller-optimal information policy policies and connect these results to the induced demand elasticities. I then characterize the set of welfare outcomes and show that total surplus varies monotonically along the Pareto frontier and is lowest under seller optimal information policy. The section concludes by showing how the obtained results continue to be useful even when outside information is not is not monotone non-overlapping: the relaxed program then characterizes a robust version of the welfare set.

Posterior Mean Reformulation

The platform's problem requires solving a maximization problem under mean-preserving spread constraints on the distribution over posteriors, making the constraint set analytically cumbersome to work with. To make the problem tractable, I consider a relaxed program that represents these constraints in terms of the induced distributions of posterior means. This relaxation is motivated by the fact that only the posterior means are payoff-relevant in my setting. Exactness of this reformulation is ensured under the maintained assumption that outside information is monotone non-overlapping as will be discussed shortly.

Original Program:

$$\max_{\tau^D \in \Delta(\Delta)(V)} f(\pi, CS) \qquad \text{s.t} \quad F_0^\delta \underset{MPS}{\geq} \tau^D \underset{MPS}{\geq} \tau^O$$

The platform chooses a distribution over posteriors, τ^D, that is less informative than full information but more informative than outside information. In the relaxed program the platform instead chooses the posterior mean distribution G^D that lies between the posterior mean distribution associated to outside information and the prior in the mean-preserving spread order:

Relaxed Program:

$$\max_{G^D} f(\pi, CS) \qquad \text{s.t} \quad F_0 \underset{MPS}{\geq} G^D \underset{MPS}{\geq} G^O$$

The reason this is a relaxed program is that τ^D being a mean-preserving spread of τ^O implies the same ordering between their posterior mean distributions, but the converse does not generally hold.[10] The following result formalizes that the mean-preserving-spread relation between posterior distributions implies the same ordering between their induced posterior-mean distributions.

Proposition 1. *If $\tau \underset{MPS}{\geq} \tau'$ then $G_\tau \underset{MPS}{\geq} G_{\tau'}$.*

Proof. This result is standard but included for completeness see Appendix. ▼ □

Hence, any feasible τ^D under the original program yields a feasible G^D under the relaxed one. However, the reverse direction does not always hold.

Representing information structures by their induced distributions of posterior means is standard in information-design problems without outside information (e.g., Kolotilin, 2018; Gentzkow and Kamenica, 2016). When considering the relaxed program, we are relaxing two constraints. Relaxing the constraint that the platform can at most fully reveal the valuation continues to be without loss here for the same reasons that appeared in the literature.[11] By contrast, relaxing the constraint coming from outside information can fail in general, but is valid under the maintained assumption that outside information is monotone non-overlapping. Under this assumption, any posterior mean distribution satisfying the mean-preserving spread constraint has a corresponding distribution over posteriors that also does (Ennuschat, 2025).

Formally,

Proposition 2. *Let τ^O be monotone non overlapping. Take $G \underset{MPS}{\geq} G_{\tau^O}$, where G_{τ^O} is the posterior mean distribution induced by τ^O. Then there exists τ^D such that $\tau^D \underset{MPS}{\geq} \tau^O$ and $G_{\tau^D} = G$.*

[10] I thank Gabriel Carroll for providing an example that illustrates this point.

[11] This is shown formally in Lemma 4.

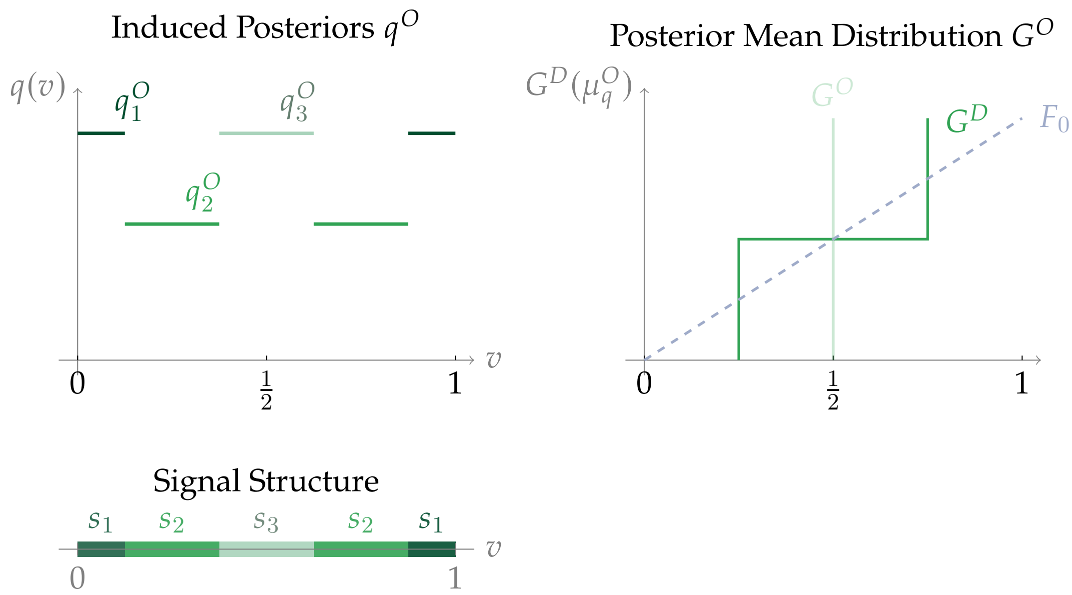

Figure 2: Illustration of Example 1

The signalling structures illustrated above induce the same posterior mean distribution as an uninformative signal. Thus signalling structures that are ranked by informativeness can induce the same posterior mean distribution.

To illustrate why the relaxation can fail without the monotone-non-overlapping assumption, consider the following example:

Example 1. *Let v be distributed uniformly over the unit interval. Suppose there is no outside information. The associated distribution over posteriors τ^N puts probability 1 on a posterior corresponding to the prior and the posterior mean G^N is degenerate at $1/2$. Now introduce outside information and consider the following signalling structure $\mathcal{S}^O$ for outside information: The buyer learns whether her valuation lies in $[0, \frac{1}{8}) \cup [\frac{7}{8}, 1]$, in $[\frac{1}{8}, \frac{3}{8}) \cup [\frac{5}{8}, \frac{7}{8})$ or in $[\frac{3}{8}, \frac{5}{8})$. Figure 2 illustrates the posteriors q^O and the posterior mean distribution G^O this induces.*

Notice that although the buyer is partially informed, the posterior mean distribution coincides with the posterior mean distribution of a completely uninformed buyer, the platform's ability to shift the buyer's beliefs differs. To see this, suppose the platform wanted to induce a posterior mean distribution that assigns probability one half to $(\frac{1}{4}$ and $\frac{3}{4})$ each. When facing uninformed buyers the signalling structure inducing this is straightforward: Tell the buyer when her valuation is below or above the mean. Under the outside information described above, it is not possible to induce the target posterior mean distribution: Take a buyer who knows their valuation lies in $[\frac{3}{8}, \frac{5}{8}]$. Any beliefs she can form must remain *within this interval, but the target posterior mean distribution is only outside the support and posteriors cannot have means outside their support.*

This illustrates that not all posterior mean distribution satisfying a mean-preserving spread constraint can be induced through the provision of additional information.

In Ennuschat (2025) I show that the assumption on outside information is not only sufficient but also necessary. Since Assumption 1 imposes that outside information is monotone-non overlapping, considering the relaxed program is without loss in this setting.

Under the relaxed program, the constraints can be restated using stochastic dominance constraints on the posterior mean distributions:

$$\max_{G^D} f(\pi, CS)$$

$$s.t. \forall x \in [0,1]: \quad \int_0^x G^D(w)dw \geq \int_0^x G^O(w)dw$$

$$\int_0^x F_0(w)dw \geq \int_0^x G^D(w)dw \qquad \text{with equality for } x = 1$$

Call G *admissible* whenever it satisfies the two stochastic dominance constraints above.

For the remainder of this paper I work with the posterior mean distribution G directly, unless clearly stated. This will allow a clean characterization of the Pareto frontier and welfare outcomes.

3.1 Pareto Frontier

Building on the previous reformulation, we can turn to analysing the platforms' information policy. First, I consider a platform that maximizes consumer surplus while ensuring a certain level of profits to the seller and then ask which such policy yields the highest overall consumer surplus. To formalize this idea, take some admissible information policy and consider whether the platform can use a different information policy that leaves seller profits unchanged but improves consumer welfare. When maximizing consumer surplus the platform has to trade-off two effects: For a given price, a better informed buyer makes better purchasing decisions. Consequently -at a given price- a platform maximizing consumer surplus would fully reveal what she knows to buyers. Prices are however not fixed and the seller reacts to the platform's information provision. Therefore the platform needs to take the effect of changing information provision on prices into account. It may provide information so that lower prices become incentive compatible for the seller.

The following theorem describes how consumer surplus is maximized while maintaining the seller's profits. It provides an explicit construction of an admissible, Pareto-improving posterior-mean distribution G^{UE}:

Theorem 1 (Pareto Frontier)**.** *Consider an admissible posterior mean distribution $\tilde{G}$. Let $\tilde{\pi} = \pi(p^*(\tilde{G}), \tilde{G})$. Take*

$$G^{UE}_{\tilde{\pi}} = \begin{cases} F_0(\omega) & \text{if } 0 \leq \omega < c_1 \\ F_0(c_1) & \text{if } c_1 \leq \omega < c_2 \\ 1 - \frac{\tilde{\pi}}{\omega} & \text{if } \omega \in [x_i, c(x_i)) \\ G^O(\omega) & \text{if } \omega \geq c_2 \text{ and } \omega \notin [x_i, c(x_i)) \end{cases}$$

where

$$c_1 = F_0^{\leftarrow}\left(1 - \frac{\tilde{\pi}}{c_2}\right)$$

$$c_2 = \min\left\{\omega \in [\tilde{\pi}, p^*(\tilde{G})] \mid \int_0^{c_1} F_0(\omega)d\omega + (c_2 - c_1)F_0(c_1) \geq \int_0^{c_2} G^O(\omega)d\omega\right\}$$

$$x_i \in X = \left\{\omega \in [0,1] \mid G^O(\omega) = 1 - \frac{\tilde{\pi}}{\omega} \text{ and } \partial_+ G^O(\omega) < \frac{\tilde{\pi}}{(\omega)^2}\right\} \cup \left\{\tilde{\pi} \mid \text{ if } G^O(\tilde{\pi}) = 0\right\}$$

$$c(x_i) = \min\left\{\omega \in [x_i, 1] \mid \int_0^{c(x_i)} G^{UE}_{\tilde{\pi}}(\omega)d\omega = \int_0^{c(x_i)} G^O(\omega)d\omega\right\}$$

Then

i) G^{UE} is admissible,

ii) profits under $G^{UE}$are the same as under $\tilde{G}$

$$\pi\left(p^*\left(G^{UE}_{\tilde{\pi}}\right), G^{UE}_{\tilde{\pi}}\right) = \tilde{\pi}$$

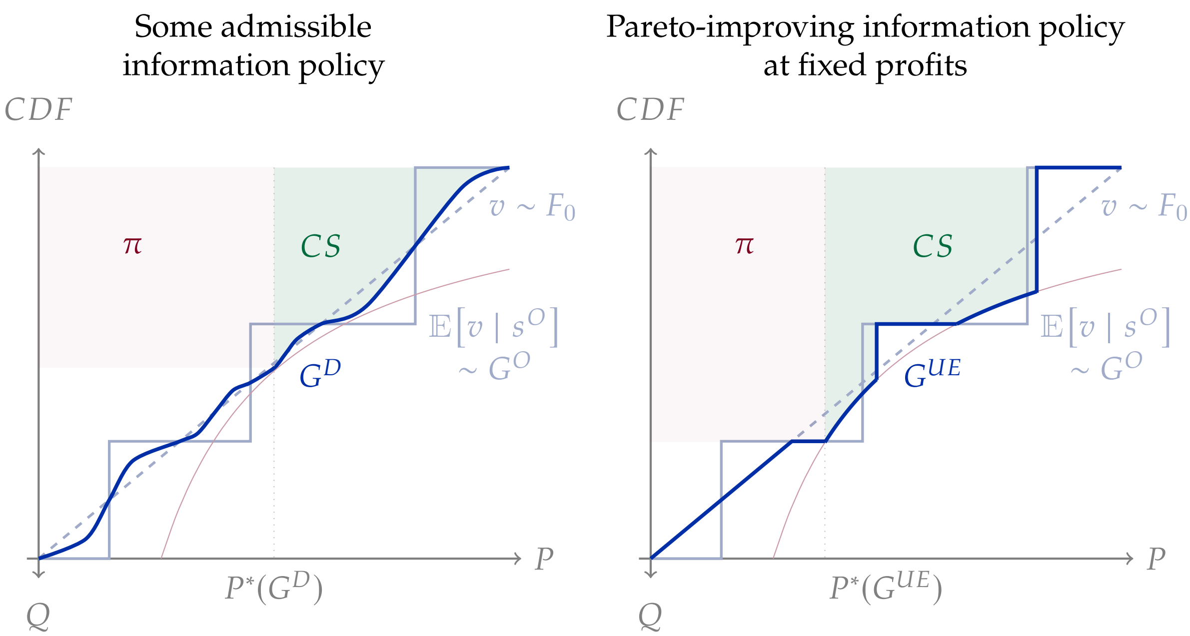

Figure 3: Pareto Frontier

The left-hand side shows an admissible induced CDF of valuations under some information policy. The right hand side displays the constructed G^{UE} from Proposition 1, which keeps seller profits at $\tilde{\pi}$ but (weakly) increases consumer surplus.

iii) and consumer surplus under G^{UE} is weakly higher than that under $\tilde{G}$:

$$\int_{p^*(G^{UE}_{\tilde{\pi}})}^{1} \left(\omega - p^*\left(G^{UE}_{\tilde{\pi}}\right)\right) dG^{UE}(\omega) \geq \int_{p^*(\tilde{G})}^{1} \left(\omega - p^*\left(\tilde{G}\right) d\tilde{G}(\omega)\right)$$

▼

where $F_0^{\leftarrow}$ is the quantile function:

$$F_0^{\leftarrow}(u) \;=\; \inf\{\, x \in \mathbb{R} : F_0(x) \geq u \,\}, \qquad u \in [0,1]$$

For any profit level attainable under some admissible information policy, there exists an alternative policy of the described form that delivers the same profits but weakly higher consumer surplus. We can compare the alternative policy to the original one. Figure 3 shows that the Pareto-improving posterior mean distribution (LHS) leads to lower prices and higher demand than the original posterior mean distribution (RHS).

The constructed policy G^{UE} can be described by three elements: the prior F_0, posterior means under outside information G^O and the isoprofit curves of the seller. Figure 3 illustrates how these elements determine the shape of G^{UE}. At valuations below c_1, the induced demand function G^{UE} coincides with the prior F_0 (full revelation). G^{UE} assigns no mass to valuations between c_1 and c_2; that is, no buyer holds expectations in this interval. For valuations above c_2, G^{UE} either coincides with posterior means under outside information or the isoprofit curve (partial revelation). In equilibrium, the seller best-responds to G^{UE} by setting a price $p = c_2$.

Partial revelation at valuations above c_2 arises whenever, under outside information, the seller would have an incentive to deviate to higher prices. The role of full revelation at the bottom is to encourage the seller to adopt a lower price: when buyers with very low valuations are revealed, those who do not receive such a signal rationally infer that their valuation is at least moderate. This inference makes them more willing to purchase at prices just below the current level. The larger set of buyers who would buy at these lower prices makes it profitable for the seller to move the price down to c_2 while keeping overall profits constant. Demand therefore rises while profits remain unchanged. With zero marginal costs, this higher demand increases total surplus, and since profits are fixed, consumer surplus increases as well.

Buyer-Optimal Information Policy Having characterized Pareto-improving policies, we now turn to the case in which the platform maximizes consumer surplus without maintaining any profit constraint for the seller. By Proposition 1 any information policy that maximizes consumer surplus at a given profit level must lie in the family $\mathcal{G}^{UE}$. Hence the overall maximizer of consumer surplus must also belong to this class. The following Proposition establishes that consumer surplus is highest when profits are minimal.

Proposition 3 (Maximal Consumer Surplus)**.** *Consumer surplus is maximized at* $\min(\pi)$ *such that* G^{UE}_{π} *is admissible.* ▼

Fixing the level of profits π the consumer optimal information policy maximizes total welfare since consumer surplus is the residual of total surplus and profits. Its shape is displayed on the left hand side of Figure 4. With zero marginal costs, efficiency is fully determined by demand in equilibrium. Consider all $G \in \mathcal{G}_{UE}$ and note that c_1 decreases as we increase profits. Recall that under G^{UE} buyers learn their valuation is below c_1 and will not purchase at the equilibrium price. Consequently c_1 is inversely related to equilibrium demand. Thus, as we decrease profits, the total level of demand under $G \in \mathcal{G}_{UE}$ increases and a lower target level of profits increases efficiency. Consumer surplus is then maximal when profits are minimal. Next, I analyse what information policymaximizes profits.

Seller-Optimal Information Policy

At the other extreme of the Pareto-frontier consider an objective where the platform maximizes profits.

The effects of market power depend on the elasticity of demand, i. e. its sensitivity to changes in prices. If demand is fully elastic, a monopolist does not ration the good and prices at marginal costs just like a competitive firm. As elasticity decreases, markups increase. The platform provides information in a way that leverages this relationship and exacerbates the effects of market power. If the platform could choose any demand function it would choose one that is completely inelastic: The buyer would always purchase the good independently of the price charged. But no information policy can induce such a demand function since the information policy needs to be consistent with the prior: a rational buyer that is fed with consistently biased messages will detect the bias and correct for it. Outside information tightens this constraint further since information provision cannot contradict what the buyer already knows to be true.

The platform's ability to magnify market power is therefore restricted: demand will have an inelastic region but its size and location are constrained by what the buyer knows to be true.

The following proposition characterizes properties that any optimal information policy must satisfy:

Proposition 4 (Seller Optimal Information Policy)**.** *Any admissible G^D that maximizes profits is such that:*

i) At $c_1 = F_0^{\leftarrow}(G^D(p))$ the stochastic dominance constraint on F_0 binds:

$$\int_0^{F_0^{\leftarrow}(G(p))} F_0(t) - G^D(t)dt = 0$$

ii) G^D assigns no mass to the subinterval $\left[F_0^{\leftarrow}(G^D(p)), p\right)$

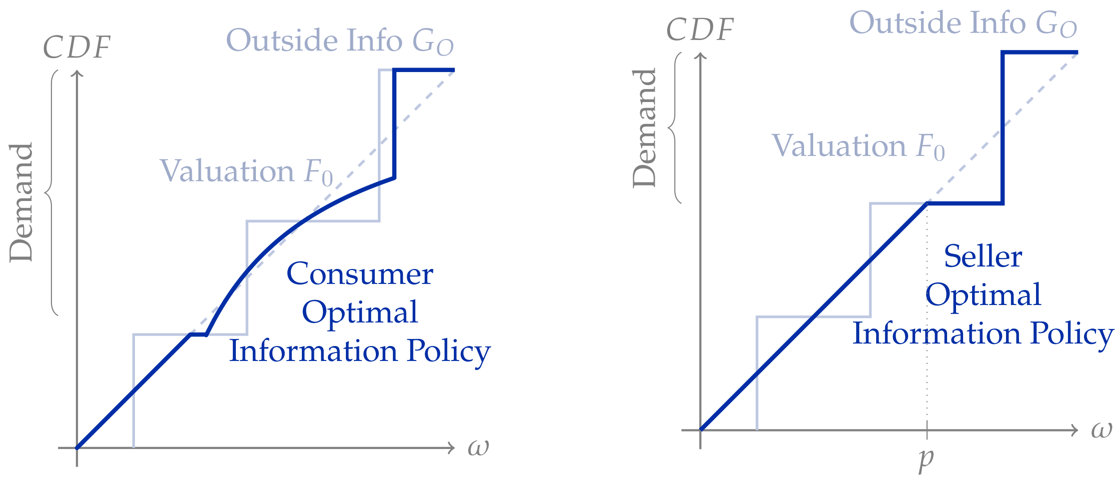

Figure 4: Buyer and Seller Optimal Information Policies

iii) G^D *is such that at* p *the stochastic dominance constraint on* G^O *binds:*

$$\int_0^p G^D(t) - G^O(t)dt = 0$$ ▼

Proposition 4 is proven in the Appendix through a sequence of lemmas establishing the optimal shape of demand for different ranges of v. These lemmas show that for and information policy that is different from Proposition 1, profits can be weakly increased by adapting the information policy so that demand remains the same at higher prices. Any information policy that is optimal must have the above properties but to find some optimal information policy it is enough to focus on the following family.

Corollary 1. *Consider the following family of information policies:*

$$G^{IE}(\omega) = \begin{cases} F_0(\omega) & \textit{if } 0 \leq \omega < c_1 \\ F_0(c_1) & \textit{if } c_1 \leq \omega < c_2 \\ G^O(\omega) & \textit{if } c_2 \leq \omega \leq 1 \end{cases}$$

where

$$c_2 = c_1 + \frac{\int_{c_1}^1 F_0(\omega)d\omega - \int_{c_2}^1 G^O(\omega)d\omega}{F_0(c_1)}$$

Denote the set of all such distribution functions $\mathcal{G}^{IE}$. Then all $G \in \mathcal{G}^{IE}$ are admissible and there is one that maximizes profits.

Figure 4 illustrates the shape of the seller-optimal information policy

Given the two primitives—the distribution of valuations F_0 and the distribution of expected valuations under outside information G^O — the seller-optimal information policy is characterized by two cutoffs c_1 and c_2. For valuations below c_1 ,just as under the buyer optimal information policy, the platform fully reveals information so that G^D coincides with the prior F_0. This ensures consistency with the prior and provides the flexibility to pool higher valuations in order to increase demand at higher prices. Between c_1 and c_2, G^D assigns no mass, again just like under the buyer optimal information policy, creating a flat segment where demand is inelastic. For valuations above c_2 there is no further revelation, so the induced demand function G^D coincides with the distribution of expected valuations under outside information G^O.

Given this information policy, the seller sets a price $p = c_2$. At this price, lowering prices slightly does not increase demand, as the flat segment makes demand unresponsive. Raising prices, on the other hand, sharply reduces demand because many consumers are exactly indifferent at c_2. Hence, the seller has no profitable local deviation.

To determine the seller-optimal rule, the platform can restrict attention to the inelastic demand functions in in $\mathcal{G}^{IE}$ and must therefore only choose the cut-off c_1. Increasing c_1 lowers demand but allows the seller to charge higher prices. The optimal c_1 balances these effects so that the induced price–demand combination lies at a point of unit elasticity, where a marginal change in demand leaves revenue unchanged. While in standard monopoly analysis unit elasticity characterizes the seller's optimal price for a given demand curve, here the platform reverses the logic: it designs the demand curve itself so that the seller's best-response price achieves unit elasticity.

By creating a range of inelastic demand, the platform amplifies the seller's market power. This structure allows the seller to charge a higher price without losing many buyers, thereby maximizing profits subject to the informational constraints.

Figure 4 displays consumer and seller optimal information policies side by side. Demand and therefore total surplus is higher under the consumer-optimal information policy. I next consider how total surplus changes when instead of optimizing the outcomes of either market side, the platform cares about a weighted average of the two.

Welfare Set

Pareto Frontier A platform that optimizes a weighted average of profits and consumer surplus—rather than focusing exclusively on one side of the market—will induce an outcome located somewhere along the Pareto frontier. Its exact location will depend on the assigned weights.

This frontier is fully characterized by information policies in Proposition 1 : we vary profits and selects a information policythat at those profits maximizes consumer surplus. We have already seen that total surplus is higher under the consumer-optimal information policy than under the seller-optimal information policy. The following proposition establishes that total surplus changes monotonically as we shift weight to the consumer.

Proposition 5 (Alignment of Total Surplus and Consumer Surplus)**.** *$\max \mathcal{CS}(\pi)+\pi$ is decreasing in π.*

Figure 5 illustrates this point: total surplus is constant along the dotted 45° lines. The logic here mirrors that of Proposition 3. Take some target profit level π and let G_π^{UE} denote the policy that maximizes consumer surplus given these profits. Now consider a decrease in the target profit level, which corresponds to a leftward shift in the target isoprofit curve. The platform has to adapt G_π^{UE} to ensure demand is weakly to the left of the new isoprofit curve everywhere. The leftward shift makes the isoprofit easier to attain allowing the platform to reveal low valuation less frequently. As fewer consumers learn that their valuation lies below the price demand increase and therefore total surplus does too.

Consequently, as profits decrease, both total and consumer surplus increase monotonically along the Pareto frontier.Under monotone non-overlapping outside information, total surplus remains constant over a range of profits and increases discretely whenever a marginal reduction in profits enables the platform to persuade an additional mass of low-expectation buyers to purchase at a lower, incentive-compatible price.

Welfare Set While the previous analysis focused on platforms that maximize a weighted average of consumer surplus and profits, the platform's objective need not take this specific form. In this section, I characterize the full set of welfare

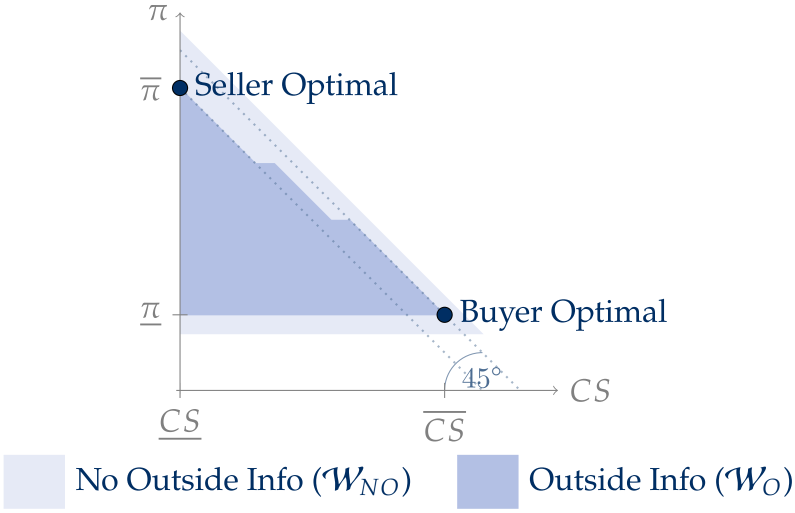

Figure 5: Welfare Outcomes

Welfare under outside information is weakly decreasing along the Pareto Frontier; the 45° degree lines outline constant total surplus levels. The welfare set under no outside information was characterized by Roesler and Szentes (2017).

outcomes that can result from any admissible information policy chosen by the platform.

Firstly, consider the seller's profits. In the preceding section, I established that the consumer-optimal information policy minimizes profits. Consequently, any profit level the platform can induce must fall within the spectrum bounded by the seller-optimal outcome ($\overline{\pi}$) and the buyer-optimal outcome ($\underline{\pi}$). The subsequent lemma confirms that all such profit levels are indeed achievable through some information policy.

Lemma 1. *For any $\pi \in [\underline{\pi}, \overline{\pi}]$ G_{π}^{UE} as defined in Theorem 1 is admissible and induces π.*
▼

For producers, this gives us all profit levels that can arise as the result of some information policy.

To also understand welfare outcomes for consumers, we need to characterize all levels of consumer surplus that are attainable by some information policy for a given level of profits. For any profit level, Proposition 3 gives us the maximal consumer surplus ($\overline{CS}(\pi)$) that some admissible information policy can induce. The following lemma characterizes minimal consumer surplus ($\underline{CS}(\pi)$) attainable by some information policy at profits π for some optimal price p. Let p^{max} denote the maximal price that- at a given information policy - is incentive compatible for the seller.

Lemma 2. *Take some $\pi \in [\underline{\pi}, \overline{\pi}]$. Then minimal consumer surplus $\underline{CS}(\pi)$ is attained by setting the highest incentive compatible price at given At those profits, consumer surplus is minimized by $G \in \mathcal{G}^{UE}$ such that*

(i) For p^{max} we have that

$$\int_0^{p^{max}} G^O(\omega)d\omega = \int_0^{p^{max}} G^D(\omega)d\omega$$

(ii) For $F_0^{\leftarrow}(G(p^{max}))$ we have that

$$\int_0^{F_0^{\leftarrow}(G(p^{max}))} G^O(\omega)d\omega = \int_0^{F_0^{\leftarrow}(G(p^{max}))} F_0(\omega)d\omega$$

▼

Revelation that minimizes consumer surplus at a given level of profits maximizes the incentive compatible price p^{max} for the seller and at that price suppresses consumer surplus to its minimal level-consumer surplus under outside information. The following lemma establishes that any level of consumer surplus $CS \in [\underline{CS}, \overline{CS}]$ can also be induced:

Lemma 3. *Any $CS(\pi) \in [\underline{CS}(\pi), \overline{CS}(\pi)]$ is implementable.* ▼

This allows us to characterize the set of welfare outcomes.

Proposition 6 (Welfare Set)**.** *The Welfare Set is given by:*

$$\mathcal{W}_O = \left\{[(\pi, \underline{CS}(\pi)), (\pi, \overline{CS}(\pi)] \mid s.t. \pi \in [\underline{\pi}, \bar{\pi}]\right\}$$

This result follows directly from Lemmas 1–3. The welfare set is generally non-convex. In standard information design problems, convexity arises because the designer can randomize across signaling structures and thereby mix welfare outcomes. Here, however, such randomization is not feasible: the platform cannot simultaneously randomize over signaling structures and over the price that the seller finds optimal given each structure. Randomizing across signalling structure typically alters the induced demand function, which in turn can change the seller's best-response in prices in a discontinuous way.

A full characterization of the welfare set would in principle require solving for all the consumer-optimal information policy at any level of profits. Leveraging the structure we can instead construct bounds that only require computing the consumer and the seller optimal information policies:

Corollary 2. *Consider the following two sets:*

$$\mathcal{W}_O^{max} = \left\{[(\pi, \underline{CS}(\underline{\pi})), (\pi, \overline{CS}(\underline{\pi}) + \underline{\pi} - \pi)] \mid s.t. \pi \in [\underline{\pi}, \bar{\pi}]\right\}$$
$$\mathcal{W}_O^{min} = \left\{[(\pi, \underline{CS}(\overline{\pi})), (\pi, \overline{CS}(\overline{\pi}) + \overline{\pi} - \pi)] \mid s.t. \pi \in [\underline{\pi}, \bar{\pi}]\right\}$$

Let $\partial\mathcal{W}$ denote the boundary of $\mathcal{W}$. Then we have that $\partial\mathcal{W}_O$ lies in between the two bounding curves $\mathcal{W}_O^{min}$ and $\mathcal{W}_O^{max}$:

$$\partial\mathcal{W}_O \subseteq \mathcal{W}_O^{max} \setminus \operatorname{int}(\mathcal{W}_O^{min})$$

Intuitively, the upper bound set $\mathcal{W}_O^{max}$ keeps total surplus fixed to its maximum level (attained in the consumer optimal outcome) and only varies how surplus is distributed. Similarly, the lower bound set $\mathcal{W}_O^{min}$ instead keeps total surplus to its minimum (attained in the seller optimal outcome) and again varies only its distribution. The true welfare frontier, $\partial\mathcal{W}_O$ lies in between the frontiers of these two sets.

Not Monotone Non-Overlapping Outside Information

Robust Welfare Outcomes When outside information is not minimal, the relaxed and unconstrained program do not coincide. Hence the solution to the relaxed program may not be implementable. The welfare set that we derived above may nevertheless be useful to make predictions of market outcomes in some robust sense:

Consider a policy maker that for a given market is concerned with the potential welfare effects that may arise as a consequence of information provision through platforms. The precision of forecasting those welfare effects depend on what we know about the platform's objective and the current market structure, shaped by

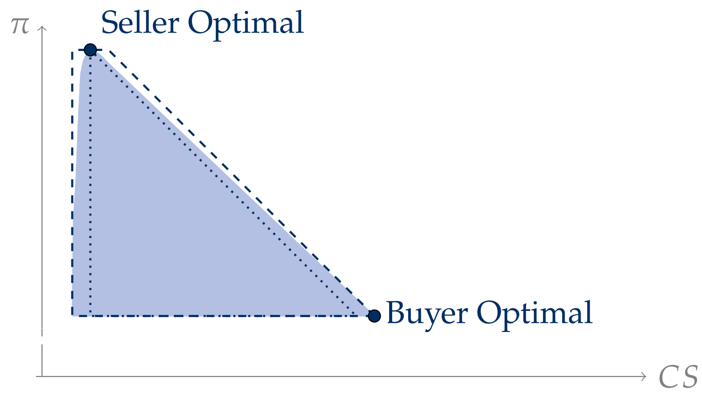

Figure 6: Robust Welfare Set and Bounds on Welfare Outcomes

outside information. Often we do not want to assume a certain objective but are instead interested in understanding the welfare consequences for any objective the platform may have. The platforms' ability to alter the current market structure depend on what consumers already know. Hence what we know about the structure of this outside information also alters our predictions of potential welfare outcomes. Suppose the policy maker does not know what exact information consumers are accessing but only observes the resulting demand function. Then to understand all potential consequences for welfare, our predictions need to be robust to any information structure that could have generated the observed demand function. Hence to characterize all welfare outcomes that could arise as a consequence of information provision through platforms, the policy maker needs to consider all those signalling structures. The following theorem establishes that the welfare set as characterized in the previous section characterizes this robust welfare set.

Theorem 2. *Let $\mathcal{W}_{G^O}$ be the set of welfare outcomes according to the relaxed problem. Let $\mathcal{W}_{\tau^O}$ be the set of welfare outcomes of the unconstrained problem. Then*

$$\bigcup_{\tau^O \in G^O} \mathcal{W}_{\tau^O} = \mathcal{W}_{G^O}$$

where $\tau \in G^O$ if τ^O is such that the induced posterior means are distributed according to G^O.

This result is an immediate consequence of the following Lemma:

Lemma 4. *Consider* $\tau^D \in \Delta(\Delta(V))$ *and let* $G^D \in \Delta(V)$ *denote the induced distribution over expected posteriors* $E(v \mid S^D)$*. Then for any distribution* $G^O \in \Delta(V)$ *that is a mean-preserving contraction of* G^D *there exists* τ^O *inducing* G^O*, where* τ^D *is a mean-preserving spread of* τ^O*.* ▼

4 Literature

A large literature studies how information provision shapes trade and welfare in markets. The central question in this literature is how information disclosure affects price setting and competition. Two main strands can be distinguished: one analyzes disclosure to sellers, the other disclosure to consumers. Existing models typically assume that information can be freely designed and abstract away from potential constraints due to agents' initial partial information — a limitation this paper seeks to address.

When information is *revealed to consumers*, it shapes market demand and through that affects the sellers' pricing decision. In a seminal contribution, Roesler and Szentes (2017) provide a characterization of the buyer-optimal disclosure rule in a bilateral trade model. They show that lower prices rather than increased purchasing accuracy are the primary channel to enhance consumer welfare. Because uninformed buyers allow sellers to extract all surplus under no disclosure, the seller-optimal policy is straightforward. Here, disclosure governs the distribution of total welfare, but does not vary with the platforms objective since efficiency can be achieved along the Pareto Frontier. When a seller jointly decides what quality of a product to offer and discloses information to consumers, the seller chooses less-than-efficient quality differentiation and does not fully disclose the valuation to the buyer Bergemann et al. (2025b). In duopoly, information design affects the intensity of competition by altering how similar consumers perceive products to be (Armstrong and Zhou, 2022). Again, the platforms objective governs the distribution of welfare but not its level- Here, the designer leverages their ability to alter the competitiveness of the market depending on their objective.
In contrast, when allowing for outside information, total surplus varies along the Pareto Frontier and is highest under consumer optimal disclosure.

The literature studying *revelation to sellers* also explores how information affects the relationship between total and consumer surplus. When consumers know their valuations, disclosure to sellers segments the market and enables price discrimination. In monopoly, this segmentation can restore efficiency and ensure that the benefits of discrimination are allocated to consumers (Bergemann et al., 2015). In multi-product monopoly, where the seller engages in second-degree price-discrimination due to menu setting and third-degree discrimination due to segmentation, an appropriate segmentation makes both consumers and the seller better off whenever the market is inefficient without segmentation (Haghpanah and Siegel, 2023).

In competitive markets, the designers segmentation has to optimally trade-off intensified competition when firms get more accurate information with their increased ability to price discriminate (Elliot et al., 2025; Bergemann et al., 2025a). (Bergemann et al., 2025a) show that consumer surplus maximization can be attained without any cost to efficiency. Consumer and total surplus are aligned in the sense that allowing for price discrimination can achieve total and consumer surplus maximization simultaneously independent of market structure. Under outside information I show that in homogeneous product monopoly alignment takes a stronger form; total surplus decreases as we move away from consumer optimal disclosure. The mechanism is also different since I consider disclosure to consumers, not the seller.

Bergemann et al. (2025b). There is work considering receivers who are privately informed. When buyers need to voluntarily reveal their valuation to sellers, they trade-off better match quality with potential surplus extraction through price discrimination when engaging with a multi-product monopolist (Ichihashi, 2020) as well as the competitive consequences with multiple sellers (Ali et al., 2023). A distinct line of work studies mechanisms that elicit private types. Kolotilin et al. (2017) shows that when the private type of the consumer is independent of the sender's information for any mechanism that elicits the receiver's private information and discloses conditionally on the private information there is a disclosure rule that does not condition and induces the same payoffs. In my setting, outside information is not independent of the valuation. Guo et al. (2025) consider a bilateral trade setting, allowing for privately informed receivers whose type is correlated with the sender's information. They derive the optimal menu of contracts consisting of a price and an information policy when the buyers private information can be described by a binary type. Both Kolotilin et al. (2017) and Guo et al. (2025) study the elicitation of private information, which is distinct from outside information.

Under elicitation the optimal revelation of additional information can be solved separately; whereas under outside information they cannot; the designer cares about the overall induced demand function.

There is a literature studying information design under constraints. Koessler and Skreta (2023) study the information design problem where the sender is partially informed; in my setting, instead, the receiver is partially informed. Onuchic and Ray (2023), Mensch (2021) and Ivanov (2021) impose structural constraints on the information the sender can reveal and require it to be monotone partitional. Le Treust and Tomala (2019) consider a setting where communication is subject to noise not controlled by the sender. Doval and Skreta (2024) provide conditions under which persuasion with additional constraints is equivalent to a unconstrained problem with an expanded state space. Terstiege and Wasser (2020) also study buyer-optimal information design in a bilateral trade model and focus on extension-proofness; the chosen experiment must leave the seller with no profitable incentive to add further information ex post. Any extension proof buyer-optimal information structure only pools two valuations.

The present framework complements this line of research by introducing a distinct form of constraint: the receiver's outside information is correlated with value and observable to the platform, thereby linking the information design problems across consumers and limiting the designer's control over feasible disclosures.

5 Conclusion

This paper shows how a platform can influence market structure and outcomes by providing information to partially informed consumers. I simplify this complex problem by invoking results from Ennuschat (2025) that allow the platform to directly optimize over the consumer's expected valuations which determine demand.

The platform's disclosure changes the elasticity of demand depending on its objective. A platform that maximizes revenue, adopts a disclosure rule that leads to inelastic demand in an intermediate region, shaping demand in a way that increases the monopolist's market power. A platform that instead aims to maximize consumer surplus will induce unit-elastic demand in an intermediate region, making the monopolist indifferent between intermediate prices. Across all information policies, efficiency is highest in this buyer-optimal-outcome and decreases for any other objective. Thus, when consumers are partially informed, additional information does not only govern the distribution of total surplus, but also affects its level. This shows how allowing for outside information can lead to qualitatively different insights from those obtained when considering full control information design problems.

In markets where digital platforms tailor information based on users' prior knowledge, the findings highlight how the design of disclosure itself becomes a key determinant of efficiency and market power.

The qualitative difference to full control information design problems suggests that introducing outside information to other market structures featuring competition, quality differentiation or where instead of disclosing information to buyers, communication occurs to multiple sellers could lead to interesting insights. Another avenue for future work would be to consider a setting where the platform does not have full control and must provides additional information independently of the consumers outside information.

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Appendix

Proof. Since τ^D is a mean-preserving spread τ^O there exists $z^D \sim \tau^D$ and $z^O \sim \tau^O$ such that $\mathbb{E}(z^D \mid z^O) = z^O$. Let the induced posterior mean distributions be given by $\mathbb{E}(z^D) \sim G^D$ and $\mathbb{E}(z^O) \sim G^O$. By the law of iterated expectations:

$$\mathbb{E}(\mathbb{E}(z^D) \mid \mathbb{E}(z^O)) = \mathbb{E}(\mathbb{E}[\mathbb{E}(z^D \mid z^O)] \mid \mathbb{E}(z^O))$$

Since τ^D is a mean-preserving spread of τ^O

$$\mathbb{E}(\mathbb{E}[\mathbb{E}(z^D \mid z^O)] \mid \mathbb{E}(z^O)) = \mathbb{E}(\mathbb{E}[z^O] \mid \mathbb{E}(z^O)) = \mathbb{E}(z^O)$$

Since $\mathbb{E}(z^D) \sim G^D$ and $\mathbb{E}(z^O) \sim G^O$ this establishes that G^D is a mean-preserving spread of G^O □

Lemma 5. *The function $T(x) = \int_0^x G^O(t)dt$ where $G^O(t)$ is a CDF and $0 \leq x \leq 1$ is continuous.*

Proof. Take some point $c \in [a,b]$ then $T(x) - T(c) = \int_a^x G^O(t)dt - \int_a^c G^O(t)dt = \int_c^x G^O(t)dt$. Since $t \in [0,1]$, the area below the cdf T(x) is bounded to be less then 1 $\int_a^b G^O(t)dt \leq 1$ Therefore we have

$$\begin{aligned} -\int_c^x 1dt \leq \int_c^x G^O(t)dt & \leq \int_c^x 1dt \\ -(x-c) \leq \int_c^x G^O(t)dtG^O(t) & \leq (x-c)dt \end{aligned}$$

Thus $\mid T(x) - T(c) \mid \leq \mid (x-c) \mid$ and thus since $\lim_{x \to c} \mid (x-c) \mid = 0$ the limits of T(x) converge and it is therefore continuous. □

Proofs Pareto Frontier & Buyer Optimal Information Design

Proof of Proposition 1. Proposition 1 follows from subsequent Lemmas 6- 8. ▲ □

Lemma 6 (Full revelation of low valuations)**.** *Take $\tilde{G}(\omega)$ such that there is no c_1 such that for all $\omega \in [0, c_1]$ $\tilde{G}(\omega) = F_0(\omega)$. Then there exists G^D that is admissible and induces weakly higher consumer surplus.* ▲

Proof Lemma 6. Let $\tilde{p}$ and $\tilde{\pi}$ be the price and profit level induced by $\tilde{G}$ respectively.
Case 1: Consider first the case where for all $\omega \in \left[0, \tilde{p}\right]$ we have that $F_0(\omega) \geq 1 - \frac{\tilde{\pi}}{\omega}$ Then consider the following G:

$$G(\omega) = \begin{cases} F_0(\omega) & \text{if } 0 \leq \omega < c_1 \\ F_0(c_1) & \text{if } c_1 \leq \omega < c_2 \\ 1 - \frac{\pi}{\omega} & \text{if } c_2 \leq \omega < \tilde{p} \\ \tilde{G}(\omega) & \text{if } \tilde{p} \leq \omega \leq 1 \end{cases}$$

where $c_2 = \frac{\tilde{\pi}}{1 - F_0(c_1)}$ so c_2 is defined as the point where the truncated pareto-distribution has the same mass as $F_0(c_1)$. Now c_1 is defined as the smallest $x \in [0, F_0^{\leftarrow}(\tilde{G}(\tilde{p}))]$ such that $\int_0^1 G(\omega)d\omega = \int_0^1 \tilde{G}(\omega)d\omega$. To show G is well defined, I need to show c_1 is well defined.
Consider $c_1 = 0$ and note that

$$\int_0^1 G(\omega) = \int_0^{\tilde{p}} \underbrace{1 - \frac{\tilde{\pi}}{\omega}}_{\leq \tilde{G}(w) \text{by optimality of } \tilde{p}} d\omega + \int_{\tilde{p}}^1 \tilde{G}(\omega) \leq \int_0^1 \tilde{G}(\omega)d\omega$$

Consider $c_1 = F_0^{\leftarrow}(\tilde{G}(\tilde{p}))$ and note that

$$\int_0^1 G(\omega)d\omega = \underbrace{\int_0^{c_1} F_0(\omega)d\omega}_{\geq \int_0^{c_1} \tilde{G}(w)} + \underbrace{\int_{c_1}^{\tilde{p}} \underbrace{F_0(c_1)}_{=G(\tilde{p})} d\omega}_{\geq \int_{c_1}^{\tilde{p}} G(\omega)d\omega} + \int_{\tilde{p}}^1 \tilde{G}(\omega) \geq \int_0^1 \tilde{G}(\omega)d\omega$$

Since by Lemma 5 $\int_0^x G(\omega)d\omega$ is continuous and by the intermediate value theorem c_1 is well defined.

Next it is necessary to establish admissibility of G. Notice that by construction G is mean-preserving.

$\boldsymbol{F_0 \succ_{MPS} G}$ For $x \in [0, c_1]$ admissibility is immediate since $G = F_0$.

For $x \in [c_1, c_2]$ we have that $\int_0^{c_1} F_0(\omega) - F_0(\omega) + \int_{c_1}^x F_0(\omega) - F_0(c_1)d\omega \geq 0$ and therefore the majorization constraint is satisfied.

For $x \in [c_2, \tilde{p}]$ we have that

$$\int_0^{c_1} F_0(\omega) - F_0(\omega) + \int_{c_1}^{c_2} F_0(\omega) - F_0(c_1)d\omega + \int_{c_2}^x \underbrace{F_0(\omega) - \left(1 - \frac{\tilde{\pi}}{\omega}\right)}_{\geq 0 \text{by assumption of Case 1}} d\omega \geq 0$$

For $x \in [\tilde{p}, 1]$ we have that $\int_x^1 G(\omega)d\omega = \int_x^1 \tilde{G}(\omega)d\omega \geq \int_x^1 F_0(\omega)d\omega$ where the last inequality follows from admissibility of $\tilde{G}$.

$\boldsymbol{G \succ_{MPS} G^O}$ For $x \in [0, c_1]$ admissibility is immediate since $G = F_0$.

For $x \in [c_1, c_2]$ I will show that $\int_0^x G(\omega)d\omega \geq \int_0^x \tilde{G}(\omega)d\omega$. Notice that c_1 is defined as the smallest x such that

$$\int_0^{\tilde{p}} G - \tilde{G}d\omega = \int_0^{c_1} F_0(\omega) - \tilde{G}(\omega)d\omega + \int_{c_1}^{c_2} F_0(c_1) - \tilde{G}(\omega)d\omega + \int_{c_2}^{\tilde{p}} \underbrace{\left(1 - \frac{\tilde{\pi}}{\omega}\right) - \tilde{G}(\omega)d\omega}_{<0 \text{ by optimality of} \tilde{p}} = 0$$

Thus it must be that

$$\int_0^{c_1} F_0(\omega) - \tilde{G}(\omega)d\omega + \int_{c_1}^{c_2} F_0(c_1) - \tilde{G}(\omega)d\omega \geq 0$$

Now suppose for some $x \in [c_1, c_2]$

$$\int_0^{c_1} F_0(\omega) - \tilde{G}(\omega)d\omega + \int_{c_1}^{x} F_0(c_1) - \tilde{G}(\omega)d\omega \leq 0$$

Then since CDFS are weakly increasing we have that

$$\int_0^{c_1} F_0(\omega) - \tilde{G}(\omega)d\omega + \int_{c_1}^{c_2} F_0(c_1) - \tilde{G}(\omega)d\omega \leq \int_0^{c_1} F_0(\omega) - \tilde{G}(\omega)d\omega + \int_{c_1}^{c_2} F_0(c_1) - \tilde{G}(x)d\omega$$

A contradiction.

For $x \in [c_1, c_2]$ again by the definition of G $\int_0^{\tilde{p}} \tilde{G}d\omega = \int_0^{\tilde{p}} Gd\omega$ and forall ω $G(\omega) > 1 - \frac{\tilde{\pi}}{\omega}$ğ so by the same argument as in the previous case: $\int_0^x G(\omega) - \tilde{G}(\omega)d\omega \geq 0$.

For $x \in [\tilde{p}, 1]$ we have that $\int_x^1 G(\omega)d\omega = \int_x^1 \tilde{G}(\omega)d\omega \leq \int_x^1 G^O(\omega)d\omega$ where the last inequality follows from admissibility of $\tilde{G}$.

Case 2: Next consider the case where $\exists\omega \in [0, \tilde{p}]$ such that $F_0(\omega) < 1 - \frac{\pi}{\omega}$. Let t be defined as the smallest x such that $F_0(x) < 1 - \frac{\pi}{\omega}$. Consider the following G:

$$G(\omega)=\begin{cases}F_0(\omega) & \text{if } 0\le\omega<c_1\\ F_0(c_1) & \text{if } c_1\le\omega<c_2\\ 1-\frac{\pi}{\omega} & \text{if } c_2\le\omega<t\\ \tilde{G}(\omega) & \text{if } t\le\omega\le 1\end{cases}$$

where $c_2=\frac{\tilde{\pi}}{1-F_0(c_1)}$ so c_2 is defined as the point where the truncated Pareto-distribution has the same mass as $F_0(c_1)$ Now c_1 is defined as the smallest $x\in[0,t]$ such that $\int_0^1 G(\omega)d\omega=\int_0^1\tilde{G}(\omega)d\omega$ To show G is well defined, I need to show c_1 is well defined.
Consider $c_1=0$ and note that

$$\int_0^1 G(\omega)d\omega=\int_0^t \underbrace{1-\frac{\tilde{\pi}}{\omega}}_{\le\tilde{G}(w)\text{by optimality of }\tilde{p}} d\omega+\int_{\tilde{p}}^1\tilde{G}(\omega)\le\int_0^1\tilde{G}(\omega)d\omega$$

Consider $c_1=t$ and note that

$$\int_0^1 G(\omega)d\omega=\underbrace{\int_0^{c_1}F_0(\omega)d\omega}_{\ge\int_0^{c_1}\tilde{G}(\omega)d\omega}+\int_t^1\tilde{G}(\omega)\ge\int_0^1\tilde{G}(\omega)d\omega$$

Since by Lemma 5 $\int_0^x G(\omega)d\omega$ is continuous and by the intermediate value theorem c_1 is well defined.

$\boldsymbol{F_0\succ_{MPS}G}$ For $x\in[0,c_1]$ admissibility is immediate since $G=F_0$.
For $x\in[c_1,c_2]$ we have that $\int_0^{c_1}F_0(\omega)-F_0(\omega)+\int_{c_1}^x F_0(\omega)-F_0(c_1)d\omega\ge 0$ and therefore the majorization constraint is satisfied.
For $x\in[c_2,t]$ we have that

$$\int_0^{c_1}F_0(\omega)-F_0(\omega)+\int_{c_1}^{c_2}F_0(\omega)-F_0(c_1)d\omega+\int_{c_2}^x\underbrace{F_0(\omega)-\left(1-\frac{\tilde{\pi}}{\omega}\right)}_{\ge 0\text{ by definition of }t}d\omega\ge 0$$

For $x\in[t,1]$ we have that $\int_x^1 G(\omega)d\omega=\int_x^1\tilde{G}(\omega)d\omega\ge\int_x^1 F_0(\omega)d\omega$ where the last inequality follows from admissibility of $\tilde{G}$.

$\boldsymbol{G\succ_{MPS}G^O}$ For $x\in[0,c_1]$ admissibility is immediate since $G=F_0$.
For $x\in[c_1,c_2]$ I will show that $\int_0^x G(\omega)d\omega\ge\int_0^x\tilde{G}(\omega)d\omega$. Notice that c_1 is defined as the smallest x such that

$$\int_0^t G-\tilde{G}d\omega=\int_0^{c_1}F_0(\omega)-\tilde{G}(\omega)d\omega+\int_{c_1}^{c_2}F_0(c_1)-\tilde{G}(\omega)d\omega+\int_{c_2}^t\underbrace{\left(1-\frac{\tilde{\pi}}{\omega}\right)-\tilde{G}(\omega)}_{<0\text{ by optimality of}\tilde{p}}d\omega=0$$

Thus it must be that

$$\int_0^{c_1}F_0(\omega)-\tilde{G}(\omega)d\omega+\int_{c_1}^{c_2}F_0(c_1)-\tilde{G}(\omega)d\omega\ge 0$$

Now suppose for some $x\in[c_1,c_2]$

$$\int_0^{c_1}F_0(\omega)-\tilde{G}(\omega)d\omega+\int_{c_1}^{x}F_0(c_1)-\tilde{G}(\omega)d\omega\le 0$$

Then since CDF's are weakly increasing we have that

$$\int_0^{c_1} F_0(\omega) - \tilde{G}(\omega) d\omega + \int_{c_1}^{c_2} F_0(c_1) - \tilde{G}(\omega) d\omega \leq \int_0^{c_1} F_0(\omega) - \tilde{G}(\omega) d\omega + \int_{c_1}^{c_2} F_0(c_1) - \tilde{G}(x) d\omega$$

A contradiction.
For $x \in [c_1, c_2]$ again by the definition of G $\int_0^{\tilde{p}} \tilde{G} d\omega = \int_0^{\tilde{p}} G d\omega$ and forall ω $G(\omega) > 1 - \frac{\tilde{\pi}}{\omega} \breve{g}$ so by the same argument as in the previous case: $\int_0^x G(\omega) - \tilde{G}(\omega) d\omega \geq 0$
For $x \in [\tilde{p}, 1]$ we have that $\int_x^1 G(\omega) d\omega = \int_x^1 \tilde{G}(\omega) d\omega \leq \int_x^1 G^O(\omega) d\omega$ where the last inequality follows from admissibility of $\tilde{G}$.
Higher Consumer Surplus Now notice that in both cases the optimal price under G is c_2 which by construction is smaller than $\tilde{p}$. Consequently demand and therefore efficiency is higher under G than under $\tilde{G}$. If efficiency is higher and profits are the same than it must be that consumer surplus is higher under G than under $\tilde{G}$. ▲

□

Lemma 7 (Region of Inelastic Demand). *Take $\tilde{G}$ where there exists c_1 such that $\forall \omega \in [0, c_1]$ we have that $= \tilde{G}(\omega) = F_0(\omega)$ but for $\omega \in [c_1, \frac{\tilde{\pi}}{1-F_0(c_1)}]$ $\tilde{G}(\omega) \neq F_0(c_1)$. Then there exists G^D that is admissible and induces weakly higher consumer surplus.* ▲

Proof Lemma 7. Let $\tilde{p}$ and $\tilde{\pi}$ be the price and profit level induced by $\tilde{G}$ respectively.
Case 1: Consider first the case where for all $\omega \in \left[0, \tilde{p}\right]$ we have that $F_0(\omega) \geq 1 - \frac{\tilde{\pi}}{\omega}$ Then consider the following G:

$$G(\omega) = \begin{cases} F_0(\omega) & \text{if } 0 \leq \omega < c_1^{new} \\ F_0(c_1^{new}) & \text{if } c_1^{new} \leq \omega < c_2 \\ 1 - \frac{\pi}{\omega} & \text{if } c_2 \leq \omega < \tilde{p} \\ \tilde{G}(\omega) & \text{if } \tilde{p} \leq \omega \leq 1 \end{cases}$$

where $c_2 = \frac{\tilde{\pi}}{1-F_0(c_1)}$ so c_2 is defined as the point where the truncated Pareto-distribution has the same mass as $F_0(c_1)$. Now c_1^{new} is defined as the smallest $x \in [c_1, F_0^{\leftarrow}(\tilde{G}(\tilde{p}))]$ such that $\int_0^1 G(\omega) d\omega = \int_0^1 \tilde{G}(\omega) d\omega$. Now to show G is well defined, I need to show c_1 is well defined.
Consider $c_1^{new} = c_1$ and note that

$$\int_0^1 G(\omega) = \int_0^{c_1} \underbrace{F_0(\omega) d\omega}_{=\tilde{G}(w)} + \underbrace{\int_{c_1}^{c_2} \underbrace{F_0(c_1)}_{\leq \tilde{G}(w)} d\omega}_{\text{since CDFS are increasing}} + \int_{c_2}^{\tilde{p}} \underbrace{\left(1 - \frac{\tilde{\pi}}{\omega}\right)}_{\substack{\leq \tilde{G}(w) \\ \text{by optimality of } \tilde{p}}} d\omega + \int_{\tilde{p}}^1 \tilde{G}(\omega) \leq \int_0^1 \tilde{G}(\omega) d\omega$$

Consider $c_1^{new} = F_0^{\leftarrow}(\tilde{G}(\tilde{p}))$ and note that

$$\int_0^1 G(\omega) d\omega = \underbrace{\int_0^{c_1} F_0(\omega) d\omega}_{\geq \int_0^{c_1} \tilde{G}(w)} + \underbrace{\int_{c_1}^{\tilde{p}} \underbrace{F_0(c_1)}_{=G(\tilde{p})} d\omega}_{\geq \int_{c_1}^{\tilde{p}} G(\omega) d\omega} + \int_{\tilde{p}}^1 \tilde{G}(\omega) \geq \int_0^1 \tilde{G}(\omega) d\omega$$

Since by Lemma 5 $\int_0^x G(\omega) d\omega$ is continuous and by the intermediate value theorem c_1^{new} is well defined.
Next it is necessary to establish admissibility of G. Notice that by construction G is mean-preserving.
$\boldsymbol{F_0 \succ_{MPS} G}$ For $x \in [0, c_1]$ admissibility is immediate since $G = F_0$.
For $x \in [c_1^{new}, c_2]$ we have that $\int_0^{c_1} F_0(\omega) - F_0(\omega) + \int_{c_1}^x F_0(\omega) - F_0(c_1^{new}) d\omega \geq 0$ and therefore the majorization constraint is satisfied.

For $x \in [c_2, \tilde{p}]$ we have that

$$\int_0^{c_1^{new}} F_0(\omega) - F_0(\omega) + \int_{c_1^{new}}^{c_2} F_0(\omega) - F_0(c_1)d\omega + \int_{c_2}^{x} \underbrace{F_0(\omega) - \left(1 - \frac{\tilde{\pi}}{\omega}\right)}_{\geq 0 \text{ by assumption of Case 1}} d\omega \geq 0$$

For $x \in [\tilde{p}, 1]$ we have that $\int_x^1 G(\omega)d\omega = \int_x^1 \tilde{G}(\omega)d\omega \geq \int_x^1 F_0(\omega)d\omega$ where the last inequality follows from admissibility of $\tilde{G}$.

$G \succ_{MPS} G^O$ For $x \in [0, c_1^{new}]$ admissibility is immediate since $G = F_0$.
For $x \in [c_1^{new}, c_2]$ I will show that $\int_0^x G(\omega)d\omega \geq \int_0^x \tilde{G}(\omega)d\omega$. Notice that c_1^{new} is defined as the smallest x such that

$$\int_0^{\tilde{p}} G - \tilde{G}d\omega = \int_0^{c_1^{new}} F_0(\omega) - \tilde{G}(\omega)d\omega + \int_{c_1^{new}}^{c_2} F_0(c_1^{new}) - \tilde{G}(\omega)d\omega + \int_{c_2}^{\tilde{p}} \underbrace{\left(1 - \frac{\tilde{\pi}}{\omega}\right) - \tilde{G}(\omega)}_{<0 \text{ by optimality of } \tilde{p}}d\omega = 0$$

Thus it must be that

$$\int_0^{c_1^{new}} F_0(\omega) - \tilde{G}(\omega)d\omega + \int_{c_1^{new}}^{c_2} F_0(c_1^{new}) - \tilde{G}(\omega)d\omega \geq 0$$

Now suppose for some $x \in [c_1^{new}, c_2]$

$$\int_0^{c_1^{new}} F_0(\omega) - \tilde{G}(\omega)d\omega + \int_{c_1^{new}}^{x} F_0(c_1^{new}) - \tilde{G}(\omega)d\omega \leq 0$$

Then since CDFS are weakly increasing we have that

$$\int_0^{c_1^{new}} F_0(\omega) - \tilde{G}(\omega)d\omega + \int_{c_1^{new}}^{c_2} F_0(c_1^{new}) - \tilde{G}(\omega)d\omega \leq \int_0^{c_1^{new}} F_0(\omega) - \tilde{G}(\omega)d\omega + \int_{c_1^{new}}^{c_2} F_0(c_1^{new}) - \tilde{G}(x)d\omega$$

A contradiction.
For $x \in [c_1^{new}, c_2]$ again by the definition of G $\int_0^{\tilde{p}} \tilde{G}d\omega = \int_0^{\tilde{p}} Gd\omega$ and forall ω $G(\omega) > 1 - \frac{\tilde{\pi}}{\omega}$g̃ so by the same argument as in the previous case: $\int_0^x G(\omega) - \tilde{G}(\omega)d\omega \geq 0$.
For $x \in [\tilde{p}, 1]$ we have that $\int_x^1 G(\omega)d\omega = \int_x^1 \tilde{G}(\omega)d\omega \leq \int_x^1 G^O(\omega)d\omega$ where the last inequality follows from admissibility of $\tilde{G}$.

Case 2: Next consider the case where $\exists \omega \in [0, \tilde{p}]$ such that $F_0(\omega) < 1 - \frac{\pi}{\omega}$. Let t be defined as the smallest x such that $F_0(x) < 1 - \frac{\pi}{\omega}$. Consider the following G:

$$G(\omega) = \begin{cases} F_0(\omega) & \text{if } 0 \leq \omega < c_1^{new} \\ F_0(c_1^{new}) & \text{if } c_1^{new} \leq \omega < c_2 \\ 1 - \frac{\pi}{\omega} & \text{if } c_2 \leq \omega < t \\ \tilde{G}(\omega) & \text{if } t \leq \omega \leq 1 \end{cases}$$

where $c_2 = \frac{\tilde{\pi}}{1 - F_0(c_1^{new})}$ so c_2 is defined as the point where the truncated Pareto-distribution has the same mass as $F_0(c_1^{new})$. Now c_1^{new} is defined as the smallest $x \in [c_1, t]$ such that $\int_0^1 G(\omega)d\omega = \int_0^1 \tilde{G}(\omega)d\omega$. To show G is well defined, I need to show c_1^{new} is well defined.

Consider $c_1^{new} = c_1$ and note that

$$\int_0^1 G(\omega) = \int_0^{c_1} \underbrace{F_0(\omega)d\omega}_{=\tilde{G}(w)} + \int_{c_1}^{c_2} \underbrace{F_0(c_1)\,d\omega}_{\substack{\leq \tilde{G}(w) \\ \text{since CDFS are increasing}}} + \int_{c_2}^{\tilde{p}} \underbrace{\left(1 - \frac{\tilde{\pi}}{\omega}\right)}_{\substack{\leq \tilde{G}(w) \\ \text{by optimality of } \tilde{p}}} d\omega + \int_{\tilde{p}}^1 \tilde{G}(\omega) \leq \int_0^1 \tilde{G}(\omega)d\omega$$

Consider $c_1^{new} = t$ and note that

$$\int_0^1 G(\omega)d\omega = \underbrace{\int_0^{c_1^{new}} F_0(\omega)d\omega}_{\geq \int_0^{c_1^{new}} \tilde{G}(\omega)d\omega} + \int_t^1 \tilde{G}(\omega) \geq \int_0^1 \tilde{G}(\omega)d\omega$$

Since by Lemma 5 $\int_0^x G(\omega)d\omega$ is continuous and by the intermediate value theorem c_1^{new} is well defined.
$\boldsymbol{F_0 \succ_{MPS} G}$ For $x \in [0, c_1^{new}]$ admissibility is immediate since $G = F_0$.
For $x \in [c_1^{new}, c_2]$ we have that $\int_0^{c_1} F_0(\omega) - F_0(\omega) + \int_{c_1^{new}}^x F_0(\omega) - F_0(c_1^{new})d\omega \geq 0$ and therefore the majorization constraint is satisfied.
For $x \in [c_2, t]$ we have that

$$\int_0^{c_1^{new}} F_0(\omega) - F_0(\omega) + \int_{c_1^{new}}^{c_2} F_0(\omega) - F_0(c_1^{new})d\omega + \int_{c_2}^x \underbrace{F_0(\omega) - \left(1 - \frac{\tilde{\pi}}{\omega}\right)}_{\geq 0 \text{ by definition of } t} d\omega \geq 0$$

For $x \in [t, 1]$ we have that $\int_x^1 G(\omega)d\omega = \int_x^1 \tilde{G}(\omega)d\omega \geq \int_x^1 F_0(\omega)d\omega$ where the last inequality follows from admissibility of $\tilde{G}$.

$\boldsymbol{G \succ_{MPS} G^O}$ For $x \in [0, c_1^{new}]$ admissibility is immediate since $G = F_0$.
For $x \in [c_1^{new}, c_2]$ I will show that $\int_0^x G(\omega)d\omega \geq \int_0^x \tilde{G}(\omega)d\omega$. Notice that c_1^{new} is defined as the smallest x such that

$$\int_0^t G - \tilde{G}d\omega = \int_0^{c_1^{new}} F_0(\omega) - \tilde{G}(\omega)d\omega + \int_{c_1^{new}}^{c_2} F_0(c_1^{new}) - \tilde{G}(\omega)d\omega + \int_{c_2}^t \underbrace{\left(1 - \frac{\tilde{\pi}}{\omega}\right) - \tilde{G}(\omega)}_{<0 \text{ by optimality of } \tilde{p}} d\omega = 0$$

Thus it must be that

$$\int_0^{c_1^{new}} F_0(\omega) - \tilde{G}(\omega)d\omega + \int_{c_1^{new}}^{c_2} F_0(c_1^{new}) - \tilde{G}(\omega)d\omega \geq 0$$

Now suppose for some $x \in [c_1^{new}, c_2]$

$$\int_0^{c_1^{new}} F_0(\omega) - \tilde{G}(\omega)d\omega + \int_{c_1^{new}}^x F_0(c_1^{new}) - \tilde{G}(\omega)d\omega \leq 0$$

Then since CDF's are weakly increasing and $F_0(c_1^{new})$ is constant we have that

$$\int_0^{c_1^{new}} F_0(\omega) - \tilde{G}(\omega)d\omega + \int_{c_1^{new}}^{c_2} F_0(c_1^{new}) - \tilde{G}(\omega)d\omega \leq \int_0^{c_1^{new}} F_0(\omega) - \tilde{G}(\omega)d\omega + \int_{c_1^{new}}^{c_2} F_0(c_1^{new}) - \tilde{G}(x)d\omega$$

A contradiction.
For $x \in [c_1^{new}, c_2]$ again by the definition of G $\int_0^{\tilde{p}} \tilde{G}d\omega = \int_0^{\tilde{p}} Gd\omega$ and forall ω $G(\omega) > 1 - \frac{\tilde{\pi}}{\omega}\breve{g}$ so by the same

argument as in the previous case: $\int_0^x G(\omega) - \tilde{G}(\omega)d\omega \geq 0$

For $x \in [\tilde{p}, 1]$ we have that $\int_x^1 G(\omega)d\omega = \int_x^1 \tilde{G}(\omega)d\omega \leq \int_x^1 G^O(\omega)d\omega$ where the last inequality follows from admissibility of $\tilde{G}$.

Higher Consumer Surplus Now notice that in both cases the optimal price under G is c_2 which by construction is smaller than $\tilde{p}$. Consequently demand and therefore efficiency is higher under G than under $\tilde{G}$. If efficiency is higher and profits are the same than it must be that consumer surplus is higher under G than under $\tilde{G}$. ▲

□

Lemma 8. *Consider a market outcome $(\tilde{p}, D(\tilde{p}))$ and let $\tilde{\pi}$ and $\tilde{CS}$ be the associated profit level and consumer surplus. Take*

$$G^{UE} = \begin{cases} F_0(\omega) & \text{if } 0 \leq \omega < c_1 \\ F_0(c_1) & \text{if } c_1 \leq \omega < c_2 \\ 1 - \frac{\tilde{\pi}}{\omega} & \text{if } \omega \in [x_i, c(x_i)) \\ G^O(\omega) & \text{if } \omega \geq c_2 \text{ and } \omega \notin [x_i, c(x_i)) \end{cases}$$

where

$$\begin{aligned} c_1 &= F_0\left(1 - \frac{\tilde{\pi}}{c_2}\right) \\ c_2 &= \min\left\{\omega \in [\pi, \tilde{p}] \mid \int_0^{c_1} F_0(\omega) + d\omega(c_2 - c_1)F_0(c_1) \geq \int_0^{c_2} G^O(\omega)d\omega\right\} \\ x_i \in X &= \left\{\omega \in [0,1] \mid G^O(\omega) = 1 - \frac{\tilde{\pi}}{\omega} \text{ and } \partial_+ G^O(\omega) < \frac{\tilde{\pi}}{(\omega)^2}\right\} \cup \{\tilde{\pi} \mid \text{ if } G(\tilde{\pi}) = 0\} \\ c(x_i) &= \min\left\{\omega \in [\pi, \tilde{p}] \mid \int_0^{c(x_i)} G(\omega)d\omega = \int_0^{c(x_i)} G^O(\omega)d\omega\right\} \end{aligned}$$

Proof. First we need to show to c_2, x_i and $c(x_i)$ are well defined. To see that c_2 is well defined, we use the intermediate value theorem. Take $c_2 = \pi$

$$\begin{aligned} c_2 = \tilde{\pi} : &\quad \int_0^{\tilde{\pi}} G(\omega)d\omega = 0 && \leq \int_0^{\tilde{\pi}} G^O(\omega)d\omega \\ c_2 = \tilde{p} : &\quad \int_0^{\tilde{p}} G(\omega)d\omega = \underbrace{\int_0^{c_1} F_0(\omega)d\omega + (\tilde{p} - c_1)F_0(c_1)}_{=\int_0^{\tilde{p}} \tilde{G}(\omega)d\omega} && \geq \int_0^{\tilde{\pi}} G^O(\omega)d\omega \end{aligned}$$

where the last inequality follows from the fact that if $(\tilde{p}, D(p))$ is a market outcome there is an admissible $\tilde{G}$ of the above form that by Lemma 6 and 7. Notice that if $G^O(\pi) = 0$ then $c_2 = \pi$. Otherwise the majorization constraint will bind at c_2. To see that x_i is well defined notice that X is a finite and ordered set. X is the set of all starting points of intervals where the isoprofit curve is above the demand function under outside information Let $x_1 = \min X$ and let $x_{i+1} = \min X \backslash \{x_1, \ldots x_i\}$. I will show that $c(x_i)$ is well defined inductively. Take $x_i = x_1$. Use the intermediate value theorem to see that $c(x_1)$ is well defined.

$c(x_1) = x_1 + \epsilon$: if $x_1 > \pi$ then

$$\underbrace{\int_0^{c_1} F_0(\omega)d\omega + \int_{c_1}^{c_2} F_0(c_1)d\omega}_{=\int_0^{c_2} G^O(\omega)d\omega} + \int_{c_2}^{x_1} G^O(\omega)d\omega \int_{x_1}^{x_1+\epsilon} \underbrace{1 - \frac{\tilde{\pi}}{\omega}}_{\substack{\geq G^O(\omega) \\ \text{for some } \epsilon > 0 \text{ since } x_1 \in X}} d\omega > \int_0^{x_1+\epsilon} G^O(\omega)d\omega$$

If $x_1 = \tilde{\pi}$

$$\int_{\tilde{\pi}}^{\tilde{\pi}+\epsilon} \underbrace{1 - \frac{\tilde{\pi}}{\omega}}_{\substack{\geq G^O(\omega) \\ \text{for some } \epsilon>0 \text{ since } x_1 \in X}} d\omega \quad > \int_0^{\tilde{\pi}+\epsilon} G^O(\omega) d\omega$$

Take $c(x_1) = 1$

$$\int_0^1 G(\omega)d\omega = \begin{cases} \overbrace{\int_0^{c_2} G(\omega)d\omega}^{<\int_0^{c_2} G^O(\omega)d\omega} + \int_{c_2}^{x_1} G^O(\omega)d\omega + \overbrace{\int_{x_1}^1 1 - \frac{\tilde{\pi}}{\omega}d\omega}^{\leq \int_{x_1}^1 \tilde{G}(\omega)d\omega} & \text{if } x_1 > \pi \\ \underbrace{\int_{\tilde{\pi}}^1 1 - \frac{\tilde{\pi}}{\omega}d\omega}_{\leq \int_0^1 \tilde{G}(\omega)d\omega} & \text{if } x_1 > \pi \end{cases}$$

$$\leq \quad \int_0^1 G^O(\omega)d\omega$$

since there must be an admissible $\tilde{G}$ that induces outcome $(\tilde{p}, D(\tilde{p}))$ and thus $\tilde{G}$ must be above the associated isoprofit curve and since it is admissible for any x: $\int_x^1 \tilde{G}(\omega)d\omega \leq \int_x^1 G^O(\omega)d\omega$

Now by induction suppose for x_i $c(x_i)$ is such that $\int_0^{c(x_i)} G(\omega)(\omega)d\omega = \int_0^{c(x_i)} G^O(\omega)(\omega)d\omega$ Now I need to show that or x_{i+1} $c(x_{i+1})$ is well defined.

Consider $c(x_{i+1}) = x_{i+1} + \epsilon$

$$\int_0^{x_{i+1}+\epsilon} G(\omega)d\omega = \underbrace{\int_0^{c(x_i)} G(\omega)d\omega}_{=\int_0^{c(x_i)} G^O(\omega)d\omega} + \int_{c(x_i)}^{x_{i+1}} G^0(\omega)d\omega + \int_{x_1}^{x_1+\epsilon} \underbrace{1 - \frac{\tilde{\pi}}{\omega}}_{\substack{\geq G^O(\omega) \\ \text{for some } \epsilon>0 \text{ since } x_{i+1} \in X}} d\omega \quad > \int_0^{x_{i+1}+\epsilon} G^O(\omega)d\omega$$

Take $c(x_{i+1}) = 1$ and apply the same argument as for x_1

$$\int_0^1 G(\omega)d\omega = \underbrace{\int_0^{c(x_i)} G(\omega)d\omega}_{=\int_0^{c(x_i)} G^O(\omega)d\omega} + \int_{c(x_i)}^{x_{i+1}} G^O(\omega)d\omega + \underbrace{\int_{x_{i+1}}^1 1 - \frac{\tilde{\pi}}{\omega}d\omega}_{\leq \int_{x_{i+1}}^1 \tilde{G}(\omega)d\omega} \leq \int_0^1 G^O(\omega)d\omega$$

since there must be an admissible $\tilde{G}$ that induces outcome $(\tilde{p}, D(\tilde{p}))$ and thus $\tilde{G}$ must be above the associated isoprofit curve and since it is admissible for any x: $\int_x^1 \tilde{G}(\omega)d\omega \leq \int_x^1 G^O(\omega)d\omega$

Next I show admissibility.

Admissibility with respect to G^O

For $x \in [0, c_2)$ by definition of c_2

For $x \in [c_2, x_1)$ If $x_1 > c_2$ then $c_2 > \tilde{\pi}$

$$\int_0^x G(\omega)d\omega = \underbrace{\int_0^{c_1} F_0(\omega)d\omega + \int_{c_1}^{c_2} F_0(c_1)d\omega}_{=\int_0^{c_2} G^\omega d\omega} + \int_{c_2}^x G^O(\omega)d\omega = \int_0^x G^O(\omega)d\omega$$

since at c_2 the majorization constraint on G^O will bind.

For $x \in [x_1, c(x_1))$ if $x_1 = \tilde{\pi}$

$$\int_0^x G(\omega)d\omega = \int_\pi^x \left(1 - \frac{\tilde{\pi}}{\omega}\right) d\omega \geq \int_0^x G^O(\omega)d\omega$$

by definition of $c(x_1)$ and if $x_1 > \tilde{\pi}$ this follows from the definition of $c(x_1)$:

$$\int_0^x G(\omega)d\omega = \int_0^{x_1} G(\omega)d\omega + \int_x^{c(x_1)} \left(1 - \frac{\tilde{\pi}}{\omega}\right) d\omega \geq \int_0^x G^O(\omega)d\omega$$

For $x \in [x_{i+1}, c(x_{i+1}))$

$$\int_0^{c(x_{i+1})} G(\omega)d\omega = \underset{\substack{=\int_{x_{i+1}}^x G^O(\omega)d\omega \\ \text{by def. of } c(x_i)}}{\int_0^{c(x_i)} G(\omega)d\omega d\omega} + \int_{c(x_i)}^{x_{i+1}} G^O(\omega)d\omega + \underbrace{\int_{x_{i+1}}^x 1 - \frac{\tilde{\pi}}{\omega} d\omega}_{\substack{\geq \int_{x_{i+1}}^x G^O(\omega)d\omega \\ \text{by def. of } c(x_{i+1})}}$$

Admissibility with respect to F_0
For $x \in [0, c_1)$ since $G = F_0$
For $x \in [c_1, c_2)$ $\int_0^x G(\omega)d\omega = \int_0^{c_1} F_0(\omega)d\omega + \int_{c_1}^x F_0(c_1)d\omega \leq \int_0^x F_0(\omega)d\omega$ For $x \in [c_2, x_1)$ if $x_1 > c_2$ then $c_2 > \tilde{\pi}$

$$\int_0^x G(\omega)d\omega = \int_0^{c_2} G(\omega)d\omega + \int_{c_2}^x G^O(\omega)d\omega = \int_0^x G^O(\omega)d\omega \leq \int_0^x F_0(\omega)d\omega$$

For $x \in [x_1, c(x_1))$ if $c_2 = \tilde{\pi}$

$$\int_0^x G(\omega)d\omega = \underbrace{\int_0^x 1 - \frac{\tilde{\pi}}{\omega} d\omega}_{\leq \int_0^x \tilde{G}(\omega)d\omega} \leq \int_0^x F_0(\omega)d\omega$$

If $c_2 > \tilde{\pi}$

$$\int_0^x G(\omega) = \underbrace{\int_0^{c_2} G(\omega)d\omega}_{=\int_0^{x_1} G^O(\omega)d\omega} + \int_{c_2}^{x_1} G^O(\omega)d\omega + \int_{x_1}^x \underbrace{\left(1 - \frac{\tilde{\pi}}{\omega}\right)}_{\leq \tilde{G}(\omega)} d\omega \leq \int_0^x \tilde{G}(\omega)d\omega \leq \int_0^x F_0(\omega)d\omega$$

For $x \in [c(x_i), x_{i+1})$

$$\int_0^x G(\omega) = \underbrace{\int_0^{c(x_i)} G(\omega)d\omega}_{=\int_0^{c(x_i)} G^O(\omega)d\omega} + \int_{c(x_i)}^x G^O(\omega)d\omega = \int_0^x G^O(\omega)d\omega \leq \int_0^x F_0(\omega)d\omega$$

For $x \in [x_i, c(x_i))$

$$\int_0^x G(\omega) = \underbrace{\int_0^{x_i} G(\omega)d\omega}_{=\int_0^{x_i} G^O(\omega)d\omega} + \int_{x_i}^x \underbrace{\left(1 - \frac{\tilde{\pi}}{\omega}\right)}_{\leq \tilde{G}(\omega)} d\omega \leq \int_0^x \tilde{G}(\omega)d\omega \leq \int_0^x F_0(\omega)d\omega$$

This establishes admissibility with respect to F_0 Thus we established admissibility of G.
Consumer Surplus:
Notice that $c_2 = p \leq \tilde{\pi}$ and hence total surplus is weakly higher under G than under $\tilde{G}$. Since by construction profits are the same under G as under $\tilde{G}$, consumer surplus must be weakly higher under G □

Proof. Note that c_1 is increasing in π. To see this, take two profits levels $\pi < \tilde{\pi}$ that can be induced by some admissible G^D. By definition of $c_2(\tilde{\pi})$

$$\int_0^{c_1(\tilde{\pi})} F_0(\omega)d\omega + (c_2(\tilde{\pi}) - c_1(\tilde{\pi}))F_0(c_1(\tilde{\pi})) \geq \int_0^{c_2(\tilde{\pi})} G^O(\omega)d\omega\}$$

Now let $\hat{c}_2(\pi)$ be defined as $\omega \in [\pi, c_2(\tilde{\pi}))$ such that

$$1 - \frac{\pi}{\hat{c}_2(\pi)} = F(c_1(\tilde{\pi}))$$

which is well defined since $1 - \frac{\pi}{\omega} > 1 - \frac{\tilde{\pi}}{\omega}$ for all $\omega \in [0,1]$ Then since $\hat{c}_2(\pi) < c_2(\tilde{\pi})$ and $G^{UE}(\tilde{)}$ is admissible

$$\int_0^{c_1(\tilde{\pi})} F_0(\omega)d\omega + (\hat{c}_2(\pi) - c_1(\tilde{\pi}))F_0(c_1(\tilde{\pi})) \geq \int_0^{\hat{c}_2(\pi)} G^O(\omega)d\omega$$

Then $\hat{c}_1 = F_0^{\leftarrow}\left(1 - \frac{\pi}{\hat{c}_2(\pi)}\right) = F_0^{\leftarrow}\left(1 - \frac{\tilde{\pi}}{c_2(\tilde{\pi})}\right) = c_1$ So the optimal $c_2(\pi) \leq \hat{c}_2(\pi)$ which implies that $c_1(\pi) \leq c_1(\tilde{\pi})$ Since c_1 is weakly increasing in π, Demand in equilibrium $D(G^{UE}_{pi}, p^*(G^{UE}_{\pi}))$ is weakly decreasing in π meaning that total surplus is weakly decreasing in π. Then if total surplus is decreasing in π since consumer surplus is the residual of total surplus and profits, consumer surplus must weakly increase in π. ▲ □

Proofs Seller Optimal Information Design

Proof of Theorem 4. Theorem 4 follows from Lemma 9- 11. ▲

□

Lemma 9 (Seller Optimal Disclosure Rule: Full revelation at the bottom)**.** *Any outcome (G^D, p) that maximizes profits is such that:*

(i) At $c_1 = F_0^{\leftarrow}(G^D(p))$ the stochastic dominance constraint on F_0 binds:

$$\int_0^{F_0^{\leftarrow}(G(\hat{p}))} F_0(t) - G(t)dt = 0$$

Proof. To show i), suppose by contradiction that G is such that $\int_0^{F_0^{\leftarrow}(t)(G(\hat{p}))} F_0(t) - G(t)dt < 0$. Then consider the following alternative $G^{\tilde{p}}_{\tilde{c}_1}$

$$G^{\tilde{p}}_{\tilde{c}_1}(t) = \begin{cases} F_0(t) & \text{if } t < \tilde{c}_1 \\ G(\hat{p}) & \text{if } \tilde{c}_1 \leq t < \tilde{p} \\ G^O(t) & \text{if } \tilde{p} \leq t \leq 1 \end{cases}$$

where $\tilde{c}_1 = \min\left\{F_0^{\leftarrow}\left(G\left(\hat{p}\right)\right)\right\}$. Let $\tilde{p}$ be the smallest $x \geq \hat{p}$ such that

$$\int_0^1 G^{\tilde{p}}_{\tilde{c}_1}(t)dt = \int_0^1 G^O(t)dt$$

Now $\tilde{p}$ is well defined by the intermediate value theorem. To see this, for $x \in [\hat{p}, 1]$ let $D(x)$ be defined as the mass difference of $G^x_{\tilde{c}_1}$ and G^O as we vary x. Notice that $\tilde{c_1} \leq \hat{p}$ since $G^O(\hat{p} \geq G(\hat{p}))$ and $\tilde{c_1} = F_0^{\leftarrow}(G(\hat{p}))$. Then $D(x)$ is given by:

$$D(x) = \int_0^1 G^x_{\tilde{c}_1}(t) - G^O(t)dt = \int_0^{\tilde{c}_1} (F_0(t) - G^O(t))dt + (x - \tilde{c}_1)G(\hat{p})t - \int_{\tilde{c}_1}^{x} G^O(t)dt$$

As shown in Lemma 5 the function $T(x) \int_a^x G^O(t)dt$ is continuous for any CDF G^O. Since $(x - \tilde{c}_1)G(\hat{p})$ is continuous and the composition of continuous functions is continuous, $D(x)$ is continuous. To apply the intermediate value theorem, consider $D(x)$ at the lower and upper end of its domain. For $x = \hat{p}$ we have

$$\begin{aligned} D(\hat{p}) &= \int_0^{\tilde{c}_1} \big(F_0(t) - G^O(t)\big)\, dt + (\hat{p} - \tilde{c}_1)G(\hat{p}) - \int_{\tilde{c}_1}^{\hat{p}} G^O(t)dt \\ &\geq \int_0^{\tilde{c}_1} (F_0(t) - G^O(t))dt + \int_{\tilde{c}_1}^{\hat{p}} G(t) - G^O(t)dt \\ &= \underbrace{\int_0^{\tilde{c}_1} (F_0(t) - G(t))dt}_{>0 \text{ by assumption}} + \underbrace{\int_0^{\hat{p}} G(t) - G^O(t)dt}_{\text{since } G \succ_{MPS} G^O} && > 0 \end{aligned}$$

On the other hand if $x = 1$

$$\begin{aligned} D(1) &= \int_0^{\tilde{c}_1} (F_0(t) - G^O(t))dt + (1 - \tilde{c}_1)F_0(\tilde{c}_1) - \int_{\tilde{c}_1}^{1} G^O(t)dt \\ &= \underbrace{\int_0^{1} (F_0(t) - G^O(t))dt}_{=0 \text{ since } F_0 \succ_{MPS} G^O} + (1 - \tilde{c}_1)F_0(\tilde{c}_1) - \int_{\tilde{c}_1}^{1} F_0(t)dt \\ &= \int_{\tilde{c}_1}^{1} F_0(\tilde{c}_1) - F_0(t)dt && \leq 0 \end{aligned}$$

By the intermediate value theorem there exists a $\tilde{p} \in (\hat{p}, 1]$ such that

$$\int_0^{\tilde{c}_1} F_0(t)dt = \int_0^{\tilde{p}} G^O(t)dt - (\tilde{p} - \tilde{c}_1)F_0(\tilde{c_1})$$

$\boldsymbol{G_{c_1}^{\tilde{p}}}$ **MPC** $\boldsymbol{F_0}$: To show $G_{c_1}^{\tilde{p}}$ is a mean preserving contraction of F_0 notice that this follows by definition of $G_{c_1}^{\tilde{p}}$: Clearly $\forall x \in [0, c_1]$ it is true since $G_{c_1}^{\tilde{p}}(x) = F_0(x)$. For $\forall x \in [c_1, \tilde{p}]$ it follows from the fact that

$$\int_0^{c_1} F_0(t) - G_{c_1}^{\tilde{p}}(t)dt + \int_{c_1}^{x} F_0(t) - G(\hat{p})dt \geq \int_0^{c_1} F_0(s) - F_0(t)dt = 0$$

where the second to last inequality holds by the definition of c_1: $G^O(c_1) = G(\hat{p})$ and cdfs are weakly increasing.
For $\forall x \in [\tilde{p}, 1]$, notice that by definition of $\tilde{p}$

$$\int_0^{\tilde{p}} G_{c_1}^{\tilde{p}}(t)dt - \int_0^{\tilde{p}} G^O(t)dt = 0$$

Thus since for any $x \in [\tilde{p}, 1]$ $G_{c_1}^{\tilde{p}}(x) = G^O(x)$ we have

$$\int_0^{x} F_0(t) - G_{c_1}^{\tilde{p}}(t)dt = \int_0^{1} F_0(t) - G^O(t)dt \geq 0$$

for all $x \in [\tilde{p}, 1]$ (The last inequality simply follows by the fact that F_0 is a MPS of G^O)

$\boldsymbol{G_{c_1}^{\tilde{p}}}$ **MPS** $\boldsymbol{G^O}$: To show $G_{c_1}^{\tilde{p}}$ is a mean preserving spread of G^O, notice $\tilde{p}$ is defined to ensure this condition. For $x \in [0, c_1]$ the condition holds since $G_{c_1}^{\tilde{p}} = F_0$ and F_0 is a mean-preserving spread of G^O:

$$\int_0^{x} G_{c_1}^{\tilde{p}}(t) - G^O(t)dt = \int_0^{x} F_0(t) - G^O(t)dt \geq 0$$

For $\forall x \in [c_1, \hat{p}]$:

$$\begin{aligned}\int_0^x G_{c_1}^{\tilde{p}}(t) - G^O(t)dt &= \int_0^{c_1} F_0(t)dt + (x - c_1)G(\hat{p}) - \int_0^x G^O(t)dt \\ &\geq \int_0^{c_1} F_0(t)dt + \int_{c_1}^x G(t) - G^O(t)dt \\ &= \underbrace{\int_0^{c_1} (F_0(t) - G(t))dt}_{\geq 0 \text{ since } F_0 \succ_{MPS} G} + \underbrace{\int_0^x G(t) - G^O(t)dt}_{\geq 0 \text{ since } G \succ_{MPS} G^O} \\ &\geq 0\end{aligned}$$

For $x \in [\hat{p}, \tilde{p}]$, notice that $\tilde{p}$ is defined to be the smallest $x \geq \hat{p}$ such that $D(x) = 0$. Thus for any $x <\leq \tilde{p}$ it must be that $D(x) \geq 0$. Since $D(x) = \int_0^x G_{c_1}^{\tilde{p}}(t) - G^O(t)dt$ it follows immediately that the majorization constraint is satisifed.
For $x \in [\tilde{p}, 1]$:

$$\begin{aligned}\int_0^x G_{c_1}^{\tilde{p}}(t) - G^O(t)dt &= \int_0^{c_1} F_0(t)dt + (\tilde{p} - c_1)G(\hat{p}) - \int_0^{\tilde{p}} G^O(t)dt + \int_{\tilde{p}}^x G^O(t) - G^O(t)dt \\ &= D(\tilde{p}) \\ &= 0\end{aligned}$$

The majorization constraint binds.
Thus $G_{c_1}^{\tilde{p}}$ is admissible.

$G_{c_1}^{\tilde{p}}$ **induces higher profits** Notice that the demand under $G_{c_1}^{\tilde{p}}$ remains constant at $(1 - G(\hat{p}))$ and by definition of $\tilde{p}$ we have that $\tilde{p} \geq \hat{p}$. Thus demand is constant at weakly higher prices and therefore profits are weakly higher. ▲

□

Lemma 10 (Seller Optimal Disclosure Rule: Inelastic Demand)**.** *Any outcome* (G^D, p) *that maximizes profits is such that:*

(ii) G^D *assigns no mass to the subinterval* $\left[F_0^{\leftarrow}(G(p)), p\right)$ ▲

Proof. Let $\tilde{p}$ be defined as follows:

$$\int_0^1 G_{c_1}^{\tilde{p}}(t)dt = \int_0^1 G(t)dt$$

which can equivalently be written as

$$\int_0^{\hat{p}} G_{c_1}^{\tilde{p}}(t) - G(t)dt = \int_{\hat{p}}^{\tilde{p}} G(t) - G_{c_1}^{\tilde{p}}(t)dt$$

now $\tilde{p} > p$ since $G(c_1) < G(\hat{p})$ and thus $\int_0^{\hat{p}} G_{c_1}^{\tilde{p}}(t)dt - \int_0^{\hat{p}} G(t)dt > 0$. $G_{c_1}^{\tilde{p}}$ is a mean-preserving spread of G and thus will also be a mean preserving spread of G^O.
Let $D(x)$ be defined as follows:

$$D(x) = \int_0^1 G_{c_1}^{\tilde{p}}(t) - G(t)dt = \int_{c_1}^{\hat{p}} F_0(t) - G(t)dt + (x - \tilde{p})G(\hat{p}) - \int_{\tilde{p}}^x G(t)dt$$

Again $D(x)$ is continuous by Lemma 5. $\tilde{p}$ is well defined by the intermediate value theorem: Consider $D(\hat{p})$

$$\begin{aligned}\int_0^1 G_{c_1}^{\tilde{p}}(t) - G(t)dt &= \int_{c_1}^{\tilde{p}} F_0(t) - G(t)dt + (\hat{p}-\tilde{p})G(\hat{p}) - \int_{\tilde{p}}^{\hat{p}} G(t)dt \\ &\geq \underbrace{\int_{c_1}^{\tilde{p}} F_0(t) - G(t)dt}_{\geq 0 \text{ since } F_0 \succ_{MPS} G} + (\hat{p}-\tilde{p})(G(\hat{p}) - G(\hat{p})) \\ &\geq 0\end{aligned}$$

Consider $D(1)$

$$\begin{aligned}D(1) &= \int_{c_1}^{\tilde{p}} F_0(t) - G(t)dt + (1-\tilde{p})G(\hat{p}) - \int_{\tilde{p}}^{1} G(t)dt \\ &= \int_{c_1}^{1} F_0(t) - G(t)dt + (1-\tilde{p})G(\hat{p}) - \int_{\tilde{p}}^{1} F_0(t)dt \\ &\leq \underbrace{\int_{0}^{1} F_0(t) - G(t)dt}_{=0 \text{ since } F_0 \succ_{MPS} G} - \underbrace{\int_{0}^{c_1} F_0(t) - G(t)dt}_{=0 \text{by definition of } G} + (1-\tilde{p}) \underbrace{(G(\hat{p}) - G^O(\tilde{p}))}_{=0 \text{ since } \tilde{p}=\min\{F_0^{\leftarrow}(G(\hat{p}))\}} = 0\end{aligned}$$

Thus by the intermediate value theorem $\tilde{p}$ is well-defined.

Notice that $G_{c_1}^{\tilde{p}}$ is mean-preserving by definition of $\tilde{p}$ since

$$\int_0^1 G_{c_1}^{\tilde{p}}(t)dt = \int_0^1 G(t)dt \qquad \implies \int_0^1 (1 - G_{c_1}^{\tilde{p}}(t))dt = \int_0^1 (1-G(t))dt$$

$\boldsymbol{F_0 \succ_{MPS} G_{c_1}^{\tilde{p}}}$: To be admissible $G_{c_1}^{\tilde{p}}$ has to be a mean-preserving contraction of F_0. For $x \in [0, c_1)$ this follows immediately from the definition of $G_{c_1}^{\tilde{p}}$ since $G_{c_1}^{\tilde{p}}(x) = F_0(x)$ For $x \in [c_1, \tilde{p})$:

$$\begin{aligned}\int_0^x F_0(t) - G_{c_1}^{\tilde{p}}(t)dt &= \int_0^{\tilde{p}} F_0(t) - F_0(t)dt + \int_{c_1}^{x} F_0(t)dt - (x-\tilde{p})G(\hat{p}) \\ &\geq \int_{\tilde{p}}^{x} \underbrace{F_0(\tilde{p})}_{=G(\hat{p})} - G(\hat{p})dt = 0\end{aligned}$$

For $x \in [\tilde{p}, 1)$:

$$\begin{aligned}\int_0^x F_0(t) - G_{c_1}^{\tilde{p}}(t)dt &= \underbrace{\int_0^{\tilde{p}} F_0(t) - G_{c_1}^{\tilde{p}}(t)dt}_{=\int_0^{\tilde{p}} F_0(t)-G(t)dt \text{ by def. of } \tilde{p}} + \int_{\tilde{p}}^{x} F_0(t) - \underbrace{G_{c_1}^{\tilde{p}}(t)}_{=G(t)}dt \\ &= \int_0^{\tilde{p}} F_0(t) - G(t)dt + \int_{\tilde{p}}^{x} F_0(t) - G(t)dt \\ &= \underbrace{\int_0^{x} F_0(t) - G(t)dt}_{\geq 0 \text{ since } F_0 \succ_{MPS} G} \\ &\geq 0\end{aligned}$$

$\boldsymbol{G_{c_1}^{\tilde{p}} \succ_{MPS} G^O}$**:** In order to show that $G_{c_1}^{\tilde{p}}$ is a mean-preserving spread of G^O, it is sufficient to show that

$G^{\tilde{p}}_{c_1}$ is a mean-preserving spread of G. For $x \in [0, \tilde{p})$ it follows from definition of $G^{\tilde{p}}_{c_1}$ and the fact that F_0 is a mean-preserving spread of G. For $x \in [\tilde{p}, \hat{p}]$

$$\begin{aligned}\int_0^x G^{\tilde{p}}_{c_1}(t) - G(t)dt &= \int_0^{\tilde{p}} F_0(t) - G(t)dt + (x - \tilde{p})G(\hat{p}) - \int_{\tilde{p}}^x G(t)dt \\ &= \int_0^{\tilde{p}} F_0(t) - G(t)dt + \int_{\tilde{p}}^x G(\hat{p}) - G(t)dt \\ &\geq \underbrace{\int_0^{\tilde{p}} F_0(t) - G(t)dt}_{\geq 0 \text{ since } F_0 \succ_{MPS} G} + \int_{\tilde{p}}^x G(\hat{p}) - G(\hat{p})dt \\ &\geq 0\end{aligned}$$

For $x \in [\hat{p}, \tilde{p})$ notice that $\tilde{p}$ is defined to be the smallest $x \geq \hat{p}$ such that $D(x) = 0$ thus for any $x \in [\hat{p}, \tilde{p})$ $D(x) > 0$ and thus by definition of $D(x)$ this immediately implies that the majorization constraint is satisfied. For $x \in [\hat{p}, \tilde{p}, 1]$

$$\int_0^x G(t) - G^{\tilde{p}}_{c_1}(t)dt = \int_0^{\tilde{p}} F_0(t) - G(t)dt + (x - \tilde{p})G(\hat{p}) - \int_{\tilde{p}}^{\tilde{p}} G(t)dt = \qquad D(\tilde{p}) = 0$$

Thus the majorization constraint binds.
$G^{\tilde{p}}_{c_1}$ induces higher profits To show profits are higher, again notice that demand is unchanged and again by definition $\tilde{p} \geq \hat{p}$. Thus profits are higher under $G^{\tilde{p}}_{c_1}$. ▲

□

Lemma 11 (Seller Optimal Disclosure Rule (iii))**.** *Any outcome (G^D, p) that maximizes profits is such that: (iii) G^D is such at p the stochastic dominance constraint on G^O binds:*

$$\int_0^{\hat{p}} G(t) - G^O(t)dt = 0$$

▲

Proof. To show iii) suppose the majorization constraint at $\hat{p}$ did not bind:

$$\int_0^{\hat{p}} G(t) - G^O(t)dt > 0$$

then consider the following alternative $G^{\tilde{p}}_{\hat{p}}$:

$$G^{\tilde{p}}_{\hat{p}}(\omega) = \begin{cases} G(\omega) & \text{if } \omega < \hat{p} \\ G(\hat{p}) & \text{if } \hat{p} \leq \omega < \tilde{p} \\ G^O(\omega) & \text{if } \tilde{p} \leq \omega \leq 1 \end{cases}$$

By the intermediate value theorem, there exists $\tilde{p} \in (\hat{p}), 1]$ such that

$$\int_0^{\hat{p}} G^{\tilde{p}}_{\hat{p}}(t) - G^O(t)dt = 0$$

since

$$\int_0^{\hat{p}} G^{\tilde{p}}_{\hat{p}}(t) - G^O(t)dt = \int_0^{\hat{p}} G(t) - G^O(t)dt > 0$$

where the inequality follows by the assumption that the majorization constraint does not bind. For $\tilde{p} = 1$ we have

$$\int_0^1 G_{\hat{p}}^{\tilde{p}}(t) - G^O(t)dt = \int_0^{\hat{p}} G(t) - G^O(t)dt + \int_{\hat{p}}^1 G(\hat{p}) - G^O(t)dt \leq \int_0^1 G(t) - G^O(t)dt = 0$$

where the last equality follows from the fact that G and G^O have the same mean. Let $\tilde{p}$ be the smallest such value.

Next we need to show that $G_{\hat{p}}^{\tilde{p}}$ is admissible.

$\boldsymbol{G_{\hat{p}}^{\tilde{p}} \succ_{MPS} G^O}$ **:** follows from the definition of $\tilde{p}$. For $x \in [0, \hat{p}]$ we have,

$$\int_0^{\hat{p}} G_{\hat{p}}^{\tilde{p}}(t) - G^O(t)dt = \int_0^{\hat{p}} G(t) - G^O(t)dt > 0$$

by assumption. Then for $x \in [\hat{p}, \tilde{p}]$ the inequality holds since $\tilde{p}$ is the smallest value such that the constraint binds.

For $x \in [\tilde{p}, 1]$ we have

$$\int_0^x G_{c_1}^{\tilde{p}}(t) - G^O(t)dt = \underbrace{\int_0^{\tilde{p}} G(t) - G^O(t)dt}_{=0 \text{ by construction of } \tilde{p}} + \int_{\tilde{p}}^x G^O(t) - G^O(t)dt = 0$$

To show $\boldsymbol{F_0 \succ_{MPS} G_{\hat{p}}^{\tilde{p}}}$ notice that for $x \in [0, \hat{p}]$ this is true since G is admissible:

$$\int_0^x F_0(t) - G_{\hat{p}}^{\tilde{p}}(t)dt = \int_0^x F_0(t) - G(t) \geq 0$$

For $x \in [\hat{p}, \tilde{p}]$

$$\int_0^x F_0(t) - G_{\hat{p}}^{\tilde{p}}(t)dt = \int_0^{\hat{p}} F_0(t) - G(t) + \int_{\hat{p}}^x F_0(t) - G(\hat{p}) > \int_0^x F_0(t) - G(t) \geq 0$$

For $x \in [\tilde{p}, 1]$ the majorization constraint holds since since $F_0 \succ_{MPS} G^O$:

$$\int_0^x F_0(t) - G_{\hat{p}}^{\tilde{p}}(t)dt = \underbrace{\int_0^{\tilde{p}} F_0(t) - G(t)dt}_{\substack{=\int_0^{\tilde{p}} F_0(t) - G^O(t)dt \\ \text{by definition of } \tilde{p}}} + \int_{\tilde{p}}^x F_0(t) - G^O(t)dt = \int_0^x F_0(t) - G^O(t)dt \geq 0$$

Thus $G_{\hat{p}}^{\tilde{p}}(t)$ is admissible and induces strictly higher profits than G rendering G suboptimal. ▲

□

Welfare Outcomes

Proof Lemma 1. Notice that it is sufficient to show that there is some G that is admissible and induces π. If there is some G, we can use Theorem 1 to obtain $G \in \mathcal{G}_{UE}$.

Case 1: $\boldsymbol{\pi} \geq \pi_{F_0}$ Consider

$$G = \begin{cases} F_0(\omega) & 0 \leq \omega < c_1 \\ F_0(c_1) & c_1 \leq \omega < c_2 \\ G^O(\omega) & c_2 \leq \omega \leq 1 \end{cases}$$

where $c_2 = \frac{\pi}{1-F_0(c_1)}$ and c_1 is defined as the largest $x \in [c_1^{SO}, F_0^{\leftarrow}(1-\pi)]$ such that $\int_0^1 G(\omega)d\omega = \int_0^1 F_0(\omega)d\omega$. c_1 is well defined by the intermediate value theorem. Take $c_1 = c_1^{SO}$

$$\begin{aligned}\int_0^1 G(\omega)d\omega &= \int_0^{c_1^{SO}} F_0(\omega)d\omega + \int_{c_1^{SO}}^{c_2} F_0(c_1^{SO})d\omega + \int_{c_2}^1 G^O d\omega \\ &= \int_0^{c_2} G^{SO}(\omega)d\omega + \underbrace{\int_{c_2}^1 G^O(\omega)d\omega}_{\geq \int_{c_2}^1 G^{SO} \text{ by admissibility of } G^{SO}} \\ &\geq \int_0^1 G^{SO} = \int_0^1 F_0\end{aligned}$$

Take $c_1 = F_0^{\leftarrow}(1-\pi)$

$$\begin{aligned}\int_0^1 G(\omega)d\omega &= \int_0^{F_0^{\leftarrow}(1-\pi)} F_0(\omega)d\omega + \int_{F_0^{\leftarrow}(1-\pi)}^1 (1-\pi)d\omega \\ &\leq \int_0^1 F_0(\omega)\end{aligned}$$

Next I show admissibility of G

Admissibility of G with respect to G^O:

$x \in [0, c_1]$ since $G = F_0$

$x \in [c_1, c_2]$ Suppose by contradiction for some x $\int_0^x G(\omega)d\omega < \int_0^x G^O(\omega)d\omega$

$$\begin{aligned}\int_0^x G(\omega)d\omega &< \int_0^x G^O(\omega)d\omega \\ \int_0^{c_1} F_0(\omega) + \int_{c_1}^x F_0(c_1)d\omega &< \int_0^x G^O(\omega)d\omega \\ \text{since CDFs are increasing: } \int_0^{c_1} F_0(\omega) + \int_{c_1}^{c_2} F_0(c_1)d\omega &< \int_0^{c_2} G^O(\omega)d\omega \\ \int_0^{c_1} F_0(\omega) + \int_{c_1}^{c_2} F_0(c_1)d\omega + \int_{c_2}^1 G^O &< \int_0^1 G^O(\omega)d\omega\end{aligned}$$

where the last line is a contradiction to the definition of c_1 and G.

$x \in [c_2, 1]$ since $G = G^O$

Admissibility of G with respect to G^O:

$x \in [0, c_1]$ since $G = F_0$

$x \in [c_1, c_2]$

$$\int_0^x G(\omega)d\omega = \int_0^{c_1} F_0(\omega)d\omega + \int_{c_1}^x F_0(c_1)d\omega leq \int_0^x F_0(\omega)d\omega$$

$x \in [c_2, 1]$ since $G = G^O$

Case 2: $\pi < \pi_{G^O}$

Let k_i be defined as all ω such that $1 - \frac{\pi}{\omega} = G^O$ and $\partial_- G^O(x \neq \frac{\pi}{x}$ so that the slope of G^O and the isoprofit curve is different at k_i. Then (k_1, k_n) are all the crossing points of G and the isoprofit curve $1 - \frac{\pi}{\omega}$ Now define G iteratively as follows:

For $\omega \in [0, k_1]$ let $G(\omega) = G^O(\omega)$.

For $\omega \in [k_i, k_{i+1}]$ where i is even and so $G(\omega) \leq 1 - \frac{\pi}{\omega}$ let $G(\omega) = 1 - \frac{\pi}{\omega}$.

For $\omega \in [k_{i+1}, k_{i+2}]$ where $(i+1)$ is odd and so $G(\omega) \geq 1 - \frac{\pi}{\omega}$ let

$$G(\omega) = \begin{cases} 1 - \frac{\pi}{\omega} & k_{i+1} \leq \omega < c(k_{i+1}) \\ G^O(\omega) & c(k_{i+1}) \leq \omega < k_{i+2} \end{cases}$$

where $c(k_{i+1})$ is defined as follows: If $\int_0^{k_{i+1}} G(\omega)d\omega + \int_{k_{i+1}}^{k_{i+2}} 1 - \frac{\pi}{\omega} d\omega \geq \int_0^{k_{i+2}} G^O(\omega)d\omega$ then let $c(k_{i+1}) = k_{i+2}$ otherwise let $c(k_{i+1})$ be defined the smallest $\omega \in [k_{i+1}, k_{i+2}]$ such that $\int_0^{k_{i+2}} G(\omega)d\omega = \int_0^{k_{i+2}} G^O(\omega)d\omega$
Take $c(k_{i+1}) = k_{i+1}$:

$$\begin{aligned} \int_0^{k_{i+2}} G(\omega)d\omega &= \underbrace{\int_0^{k_{i-1}} G(\omega)d\omega}_{\geq \int_0^{k_{i-1}} G^O(\omega)d\omega} + \int_{k_{i-1}}^{k_i} \underbrace{1 - \frac{\pi}{\omega}}_{> G^O \text{since } \omega \in [k_{i-1}, k_i]} d\omega + \int_{k_i}^{k_{i+1}} G^O(\omega)d\omega \\ &\geq \int_0^{k_{i+2}} G^O(\omega)d\omega \end{aligned}$$

Where $\int_0^{k_{i-1}} G(\omega)d\omega \geq \int_0^{k_{i-1}} G^O(\omega)d\omega$ follows from the iterative definition of $G(\omega)$.
Take $c(k_{i+1}) = k_{i+2}$.Then by assumption of this case:

$$\int_0^{k_{i+2}} G(\omega)d\omega < \int_0^{k_{i+1}} G^O(\omega)d\omega$$

Therefore $c(k_{i+1})$ is well defined by the intermediate value theorem.

Next I show admissibility of $G(\cdot)$ wrt to G^O.

Admissibility with respect to G^O
For $\omega \in [0, k_1]$ since $G(\omega) = G^O(\omega)$
For $\omega \in [k_i, k_{i+1}]$ we have that

$$\int_0^x G(\omega)d\omega = \underbrace{\int_0^{k_i} G(\omega)d\omega}_{\geq \int_0^{k_{i-1}} G^O(\omega)d\omega} + \int_{k_i}^x \underbrace{1 - \frac{\pi}{\omega}}_{> G^O \text{since } \omega \in [k_{i-1}, k_i]} d\omega \geq \int_0^x G^O(\omega)d\omega$$

For $\omega \in [k_{i+1}, c(k_{i+1})]$ suppose by contradiction that

$$\begin{aligned} \int_0^x G(\omega)d\omega &< \int_0^x G^O(\omega)d\omega \\ \underbrace{\int_0^{k_{i+1}} G(\omega)d\omega}_{\geq \int_0^{k_{i+1}} G^O(\omega)d\omega} + \int_{k_{i+1}}^x 1 - \frac{\pi}{\omega} d\omega &< \int_0^x G^O(\omega)d\omega \end{aligned}$$

Then since for $\omega \in [k_{i+1}, k_{i+2}]$ $1 - \frac{\pi}{\omega} < G^O(\omega)$ we have:

$$\int_0^{k_{i+1}} G(\omega)d\omega + \int_{k_{i+1}}^{k_{i+2}} 1 - \frac{\pi}{\omega} d\omega < \int_0^{k_{i+2}} G^O(\omega)d\omega$$

But this contradicts the definition of $c(k_{i+1})$.
For $\omega \in [c(k_{i+1}), k_{i+2}]$ we have that

$$\int_0^x G(\omega)d\omega = \underbrace{\int_0^{c(k_{i+1})} G(\omega)d\omega}_{\substack{= \int_0^{c(k_{i+1})} G^O(\omega)d\omega \\ \text{by def. of } c(k_{i+1})}} + \int_{c(k_{i+1})}^x G^O(\omega)d\omega = \int_0^x G^O(\omega)d\omega$$

Admissibility with respect to F_0
I will show admissibility with respect to F_0 by showing that G^{BO} is a mean preserving spread of G.
For $\omega \in [0, k_1]$ since $G(\omega) = G^O(\omega)$.
For $\omega \in [k_1, k_2]$ since

$$\int_0^x G(\omega)d\omega = \int_0^{k_1} G^O(\omega)d\omega + \int_{k_1}^x 1 - \frac{\pi}{\omega}d\omega \leq \int_0^{k_1} G^{BO}(\omega)d\omega + \int_{k_1}^x 1 - \frac{\pi^{BO}}{\omega}$$
$$= \int_0^x G^{BO}(\omega)d\omega$$

where the inequality of the first term follows from admissibility of $G^{BO}(\omega)$ and on the second terms it follows from the fact that $\pi^{BO} < \pi$. The last equality holds since the smallest ω where $1 - \frac{\pi^{BO}}{\omega} > G^O$ is smaller than k_1.
For $\omega \in [k_{i+1}, c(k_{i+1})]$ we have that

$$\int_0^x G(\omega)d\omega = \int_0^{k_j} G(\omega)d\omega + \int_{k_j}^x 1 - \frac{\pi}{\omega}d\omega$$

Now notice that for any k_j we have that $c(k_j) < c(k_j^{BO})$ since $1 - \frac{\pi}{\omega} < 1 - \frac{\pi^{BO}}{\omega}$

$$\int_0^x G(\omega)d\omega = \underbrace{\int_0^{k_j} G(\omega)d\omega}_{\leq \int_0^x G^{BO}(\omega)d\omega} + \int_{k_j}^x \underbrace{\left(1 - \frac{\pi}{\omega}\right)}_{\leq \left(1 - \frac{\pi^{BO}}{\omega}\right)} d\omega \leq \int_0^x G^{BO}(\omega)d\omega$$

where the inequality on the first term follows by the inductive hypothesis. For $\omega \in [c(k_{i+1}), k_{i+2}]$ we have that

$$\int_0^x G(\omega)d\omega = \int_0^{c(k_j)} G(\omega)d\omega + \int_{c(k_j)}^x G^O(\omega)d\omega = \int_0^x G^O(\omega)d\omega \leq \int_0^x G^{BO}(\omega)$$

This follows by definition of $c(k_j)$.
For $\omega \in [c(k_i), k_{i+1}]$ we have that

$$\int_0^x G(\omega)d\omega = \underbrace{\int_0^{k_i} G(\omega)d\omega}_{\leq \int_0^{k_i} G^{BO}(\omega)d\omega} + \int_{k_i}^x \underbrace{\left(1 - \frac{\pi}{\omega}\right)}_{< \left(1 - \frac{\pi^{BO}}{\omega}\right)} d\omega = \int_0^x G^{BO}(\omega)d\omega$$

where the inequality on the first term follows by the inductive hypothesis. and the last equality follows from the fact that $1 - \frac{\pi^{BO}}{\omega} > 1 - \frac{\pi}{\omega} \geq G(\omega)$ since $\omega \in [k_i, k_{i+1}]$ and therefore $G^{BO} = 1 - \frac{\pi^{BO}}{\omega}$.
Case 3: $\pi \in (\pi^{F_0}, \pi^{G^O})$ Consider $G(\omega)$ defined as follows:

$$G(\omega) = \begin{cases} G^O(\omega) & 0 \leq \omega < c_1 \\ F_0(c_2) & c_1 \leq \omega < c_2 \\ G^O(\omega) & c_2 \leq \omega \leq 1 \end{cases}$$

where c_2 is defined as the smallest ω such that $1 - \frac{\pi}{\omega} = F_0(\omega)$ which is well defined since $\pi > \pi_{F_0}$. Now let c_1 be defined such that $\int_0^1 G(\omega)d\omega = \int_0^1 G^O(\omega)d\omega$. c_1 is well- defined by the intermediate value theorem. Consider $c_1 = (G^O)^{\leftarrow}(F_0(c_2))$. Then:

$$\int_0^1 G(\omega)d\omega = \int_0^{c_1} G^O(\omega)d\omega + \int_{c_1}^{c_2} G^O(c_1)d\omega + \int_{c_2}^1 G^O(\omega)d\omega < \int_0^1 G(\omega)d\omega$$

where we know that $c_1 = (G^O)^{\leftarrow}(F_0(c_2)) \leq c_2$ since $\pi > \pi_{G^O}$ and thus $G^O > (1 - \frac{\pi}{\omega}$.
Consider $c_1 = 0$. Then:

$$\int_0^1 G(\omega)d\omega = \int_0^{c_2} F_0(c_2) + \underbrace{\int_{c_2}^1 G^O(\omega)d\omega}_{\int_{c_2}^1 F_0(\omega)d\omega} \geq \int_0^1 F_0(\omega)d\omega = \int_0^1 G^O(\omega)d\omega$$

Then by the intermediate value theorem c_1 is well-defined.
Admissibility with respect to G^O
$x \in [0, c_1) \cup [c_2, 1]$ since $G = G^O$
$x \in [c_1, c_2)$ Suppose by contradiction that

$$\int_0^x G(\omega)d\omega < \int_0^x G^O(\omega)d\omega$$
$$\int_0^{c_1} G^O(\omega)d\omega + \int_{c_1}^x F_0(c_2)d\omega < \int_0^x G^O(\omega)d\omega$$

Then we would have:

$$\int_0^{c_1} G^O(\omega)d\omega + \int_{c_1}^{c_2} F_0(c_2)d\omega < \int_0^{c_2} G^O(\omega)d\omega$$

which contradicts the definition of c_1
Admissibility with respect to F_0
$x \in [0, c_1) \cup [c_2, 1]$ since $G = G^O$
$x \in [c_1, c_2)$

$$\int_x^1 G(\omega)d\omega = +\int_x^{c_2} F_0(c_2)d\omega \int_{c_2}^1 G^O(\omega)d\omega \geq \int_x^{c_2} F_0(\omega)d\omega + \int_{c_2}^1 F_0(\omega)d\omega$$

□

Lemma 2. **(i) At p^{max} the majorization constraint on G^O binds**
Notice that by admissibility of G it must be that $\int_{p^{max}}^1 G(\omega)d\omega \leq \int_{p^{max}}^1 G^O(\omega)d\omega$ Consumer Surplus is given by:

$$\int_{p^{max}}^1 \left(\omega - p^{max}\right) dG(\omega) = \int_{p^{max}}^1 (1 - G(\omega))d\omega$$

Thus consumer surplus is minimized by letting the majorization constraint on G^O bind. To show this I will first establish that for any G that minimizes consumer surplus for a given level of profits it must be that $p^{max} \geq k$, where k is the largest $x \in [0, 1]$ such that $G^O(\omega) = 1 - \frac{\pi}{\omega}$. Consider some G such that this is not the case and let $\tilde{G}$ be defined as

$$\tilde{G}(\omega) = \begin{cases} G(\omega) & 0 \leq \omega < c \\ G(k) & k < \omega < \tilde{p}^{max} \\ G^O(\omega) & \tilde{p}^{max} \leq \omega \leq 1 \end{cases}$$

where $\tilde{p}^{max}$ is defined as $\omega \in [c, 1]$ such that $G(c) = 1 - \frac{\pi}{\omega}$ and c is defined as $\omega \in [k, G^{\leftarrow}(1 - \pi)]$ such that $\int_0^1 \tilde{G}(\omega)d\omega = \int_0^1 G(\omega)d\omega$ which is well defined by the intermediate value theorem:

Take $\tilde{p}^{max} = k$ Then we have that

$$\int_0^1 \tilde{G}(\omega)d\omega = \int_0^k G(\omega)d\omega$$

since G is admissible.
Take $\tilde{p}^{max} = G^{\leftarrow}(\omega)(1-\pi)$. Then we have that

$$\int_0^1 \tilde{G}(\omega)d\omega = \int_0^{\tilde{p}^{max}} G(\omega)d\omega + \int_{\tilde{p}^{max}}^1 G(\tilde{p}^{max})G^O(\omega)d\omega \leq \int_0^1 G(\omega)d\omega$$

Now we need to show that the $\tilde{G}$ is admissible.
Admissibility with respect to F_0
For $x \in [0, p^{max})$ since $\tilde{G}(\omega) = G(\omega)$
For $x \in [c, \tilde{p}^{max})$ we have that $\int_0^x \tilde{G}(\omega)d\omega = \int_0^c G(\omega)d\omega + \int_{p^{max}}^x G(c)d\omega \leq \int_0^x G(\omega)d\omega \leq \int_0^x F_0(\omega)d\omega$
For $x \in [\tilde{p}^{max}, 1)$ we have that $\int_x^1 \tilde{G}(\omega)d\omega = \int_x^1 G^O(\omega)d\omega \geq \int_x^1 F_0(\omega)d\omega$
Admissibility with respect to G^O
$x \in [0, c]$ since $\int_0^x \tilde{G}(\omega)d\omega = \int_0^x G(\omega)d\omega$
For $x \in [c, \tilde{p}^{max})$ Suppose that

$$\int_0^x \tilde{G}(\omega)d\omega < \int_0^x G^O(\omega)d\omega$$
$$\int_0^c G(\omega)d\omega + \int_c^x 1 - \frac{\pi}{\omega}d\omega < \int_0^x G^O(\omega)d\omega$$

Then since $c \geq k$:

$$\int_0^c G(\omega)d\omega + \int_c^{\tilde{p}^{max}} 1 - \frac{\pi}{\omega}d\omega < \int_0^{\tilde{p}^{max}} G^O(\omega)d\omega$$

which contradicts the definition of c. For $x \in [\tilde{p}^{max}, 1)$ we have that $\int_x^1 \tilde{G}(\omega)d\omega$. Thus we know that $p^{max} > k$
With this I will show that at p^{max} the majorization constraint binds. For any $\tilde{G}$ such that this constraint does not bind, we can find an admissible G that makes the constraint bind. Take some G where at p^{max} we have that the majorization constraint is slack. Then take:

$$\tilde{G}(\omega) = \begin{cases} G(\omega) & 0 \leq \omega < p^{max} \\ 1 - \frac{\pi}{\omega} & p^{max} \leq \omega \leq \tilde{p}^{max} \\ G^O(\omega) & = \tilde{p}^{max} \leq \omega \leq 1 \end{cases}$$

where $\tilde{p}^{max}$ is defined as $\omega \in [p^{max}, 1]$ such that $\int_0^1 \tilde{G}(\omega)d\omega = \int_0^1 G(\omega)d\omega$ which is well defined by the intermediate value theorem:
Take $\tilde{p}^{max} = p^{max}$ Then we have that

$$\int_0^1 \tilde{G}(\omega)d\omega = \int_0^{p^{max}} G(\omega)d\omega + \int_{p^{max}}^1 G^O(\omega)d\omega > \int_0^1 G(\omega)d\omega$$

since the majorization constraint of G on G^O is slack at p^{max}.
Take $\tilde{p}^{max} = 1$ Then we have that

$$\int_0^1 \tilde{G}(\omega)d\omega = \int_0^{p^{max}} G(\omega)d\omega + \int_{p^{max}}^1 1 - \frac{\pi}{\omega}G^O(\omega)d\omega \leq \int_0^1 G(\omega)d\omega$$

by definition of the isoprofit curve. All that is left is to show that $\tilde{G}$ is admissible.

Admissibility with respect to F_0
For $x \in [0, p^{max})$ since $\tilde{G}(\omega) = G(\omega)$
For $x \in [p^{max}, \tilde{p}^{max})$ we have that $\int_0^x \tilde{G}(\omega)d\omega = \int_0^{p^{max}} G(\omega)d\omega + \int_{p^{max}}^x 1 - \frac{\pi}{\omega}d\omega \leq \int_0^x G(\omega)d\omega \leq \int_0^x F_0(\omega)d\omega$
For $x \in [\tilde{p}^{max}, 1)$ we have that $\int_x^1 \tilde{G}(\omega)d\omega = \int_x^1 G^O(\omega)d\omega \geq \int_x^1 F_0(\omega)d\omega$
Admissibility with respect to G^O
$x \in [0, p^{max}]$ since $\int_0^x \tilde{G}(\omega)d\omega = \int_0^x G(\omega)d\omega$
For $x \in [p^{max}, \tilde{p}^{max})$ Suppose that

$$\int_x^1 \tilde{G}(\omega)d\omega = \int_x^{\tilde{p}^{max}} \underbrace{1 - \frac{\pi}{\omega}}_{\leq G(\omega)}d\omega + \underbrace{\int_{\tilde{p}^{max}}^1 G^O(\omega)d\omega}_{\int_{\tilde{p}^{max}}^1 G(\omega)d\omega}$$

we have that $\int_x^1 \tilde{G}(\omega)d\omega = \int_x^{\tilde{p}^{max}} \underbrace{1 - \frac{\pi}{\omega}}_{\leq G(\omega)}d\omega + \underbrace{\int_{\tilde{p}^{max}}^1 G^O(\omega)d\omega}_{\int_{\tilde{p}^{max}}^1 G(\omega)d\omega} \leq \int_x^1 G\omega(\omega)d\omega$ □

Proof. By Lemma 2, let $p^{\max}(\pi)$ denote the price that induces $\underline{CS}(\pi)$. Let $p^{\min} \equiv c_2$ where c_2 is defined as the c_2 of G_π^{UE} in Theorem 1. Then $p^{\min}$ induces $\overline{CS}(\pi)$.

Fix any target $CS(\pi) \in [\underline{CS}(\pi), \overline{CS}(\pi)]$ and define

$$\alpha \in [0,1] \quad \text{such that} \quad CS(\pi) = \alpha\, \underline{CS}(\pi) + (1-\alpha)\, \overline{CS}(\pi).$$

Consider the mixed pricing strategy that sets $p^{\max}(\pi)$ with probability α and $p^{\min}$ with probability $1-\alpha$. Consumer surplus is linear in the seller's randomization over prices (it is an expectation over the realized price), so the induced expected consumer surplus equals

$$\alpha\, \underline{CS}(\pi) + (1-\alpha)\, \overline{CS}(\pi) = CS(\pi).$$

Hence any level in $[\underline{CS}(\pi), \overline{CS}(\pi)]$ can be induced. □

Equivalence of Original and Relaxed Program

Proof Lemma 4 . Consider $\mu^D \sim G^D$ and $\mu^O \sim G^O$ where G^D is a mean-preserving spread of G^O. Then:

$$\begin{aligned}\mu^O &= E\left(E\left(\mu^D \mid \mu^O\right)\right)\\ &= \int_0^1 \mu^D dF_{\mu^D|\mu^O}\end{aligned}$$

where $F_{\mu^D|\mu^O}$ is the conditional distribution of μ^D given μ^O. If we think of G^O and G^D as lotteries with outcomes μ^O and μ^O respectively, then the conditional distribution function tells us how different branches of the lottery G^D are combined into one branch of the lottery G^O. Now τ^D is the two stage lottery, where the first stage describes the probability of some posterior q^D and the second stage describes the probabilities assigned to different valuations. We can construct τ^O from τ^D using $F_{\mu^D|\mu^O}$ to ensure that τ^D is a mean-preserving spread of τ^O. Define q^O as follows:

$$q^O = \int_0^1 q^D dF_{q^D|q^O}$$

where $f_{q^D|q^O} = f_{\mu^D|\mu^O} \cdot \tau^D(q^D \mid E(q^D) = \mu^D)$ and

$$f_{\mu^D|\mu^O} = \begin{cases} \frac{\partial F_{q^D|q^O}(\cdot)}{\partial q^D} & \text{whenever } \lim_{t\to(q^D)^+} F_{q^D|q^O}(t) = \lim_{t\to(q^D)^-} F_{q^D|q^O}(t) \\ \lim_{t\to(q^D)^+} F_{q^D|q^O}(t) - \lim_{t\to(q^D)^-} F_{q^D|q^O}(t) & \text{otherwise} \end{cases}$$

Let $\tau(q^O)$ be defined by the posterior mean distribution G^O:

$$\tau(q^O) = g^O(\mu^O)$$

such that $E(q^O) = \mu^O$[12], where again

$$g^O(\mu^O) = \begin{cases} \frac{\partial G^O(\mu^O)}{\partial \mu^O} & \text{whenever } \lim_{t\to(\mu^O)^+} G^O(t) = \lim_{t\to(\mu^O)^-} G^O(t) \\ \lim_{t\to(\mu^O)^+} G^O(t) - \lim_{t\to(\mu^O)^-} G^O(t) & \text{otherwise} \end{cases}$$

Think of τ^O as a four stage lottery, where the first stage is pinned down by $G^O(\mu^O)$ and the second stage tells us how we randomize over posterior means μ^D to obtain that μ^O given by $f_{\mu^D|\mu^O}$, the third stage tells us how we randomize over all posteriors q^D that induce mean μ^D, precisely $\tau^D(q^D \mid E(q^D) = \mu^D)$ and the last stage simply corresponds to the posterior q^D. Let by construction:

$$\begin{aligned} E(q^D \mid q^O) &= \int q^D d\tau(q^D \mid q^O) \\ &= \int q^D dF_{q^D|q^O} \\ &= q^O \end{aligned}$$

where the last line follows from the construction of q^O. The second to last line follows from the fact that each posterior q^O induces a different posterior mean μ^O. Then $\tau(q^D \mid q^O) = \tau(q^D \mid \mu^O)$ and we can consequently think of $\tau(q^D \mid \mu^O)$ as the three-stage lottery derived from τ^O after the realisation of the first stage. The probability of q^D is then given by the probability that mean μ^D realises in the second stage given by $f_{\mu^D|\mu^O}$ and the probability that given this mean μ^D the posterior q^D is selected: $\tau^D(q^D \mid E(q^D) = \mu^D)$, as described by the third stage. So we have that $\tau^D(q^D \mid q^O) = f_{\mu^D|\mu^O}\tau^D(q^D \mid E(q^D) = \mu^D) = f_{q^D|q^O}$ ▲ □

[12]This is well defined, since by construction of q^O there is only one q^O with mean μ^O